CSE 105: Automata and Computability Theory **Winter 2015**

Homework #6

Due: Friday, March 13th, 2015, 11:59 PM

Submit your solutions as `HW61.pdf`, `HW62.pdf` and `HW63.pdf`, using the `bundleHW6` command on ieng6.

Consider the following two languages

$$L_1 = \{\langle M \rangle \mid M \text{ is a Turing machine and } |L(M)| = 1\}$$

$$L_2 = \{\langle M \rangle \mid M \text{ is a Turing machine and } |L(M)| = 2\}$$

i.e., L_1 is the set of all Turing machines that accept precisely one input string, and L_2 is the set of Turing machines that accept precisely two inputs.

Problem 1

In this problem you will show that L_1 is neither R.E. nor co-R.E.

a. Prove that L_1 is not R.E. by reduction from the diagonal language

$$D = \{\langle M \rangle \mid M \text{ is a Turing machine such that } \langle M \rangle \notin \mathcal{L}(M)\}$$

b. Prove that L_1 is not co-R.E. by reduction from the complement of the diagonal language $\overline{D}$.

Problem 2

We say that two languages A and B are Turing equivalent if there is both a map reduction from A to B, and a map reduction from B to A. In this problem you will prove that the languages L_1 and L_2 are Turing equivalent.

a. Give a map reduction from L_1 to L_2

b. Give a map reduction from L_2 to L_1.

Hint: The b. direction is the harder one. For this direction, think about how, given a machine M, you might design a machine M' that accepts all strings M does, except the first one.

Problem 3

Use the results from problem 1 and problem 2 to prove that the language L_2 is *not* Turing equivalent to the diagonal language D.