Bayesian pre-sample estimate of the random variable $\tilde{\mu}$ is distributed as a non-informative diffuse prior. Let $g_1, \ldots, g_n$ be a random *i.i.d.* sample corresponding to the normal probability structure (23), which is drawn from a normal distribution with known precision W, but whose Bayesian pre-sample prior estimate of $\tilde{\mu}$ is a diffuse-normal distribution. With a known variance, the posterior distribution of $\tilde{\mu}$ after n independent sample observations is

$$\tilde{\mu} \sim N(\hat{g},\ 1/nW) \ . \qquad \textbf{(24)}$$

From (24) and (23), $E[g]=E[\tilde{\mu}]=\hat{g}$. Therefore, $g-E[g] = g-\hat{g}$, and, from applying definition (8) to this situation, $x=g-\hat{g}$ and $x_i=g_i-\hat{g}$.

For *given* values of W *and* $\tilde{\mu}$, the random variable g is distributed according to (23) as

$$g \sim N(\tilde{\mu},\ 1/W) \ , \qquad \textbf{(25)}$$

whereas, for any given value of W alone, the random variable $\tilde{\mu}$ is distributed according to (24). Combining these two quasi-independent realizations of normal processes, the random variable $x=g-\hat{g}$ must be distributed normally with mean zero and variance equal to the sum of the variance of the normal process (24) plus the variance of the conditionally-independent normal process (25). After adding together the two variances ($1/nW$ from (24) plus $1/W$ from (25)), the posterior distribution of $x=g-\hat{g}$ comes out to be

$$x \sim N(0,\ (n+1)/nW) \ . \qquad \textbf{(26)}$$

Thus far, the specification has proceeded as if W were known. When W is uncertain, Bayesian statistical theory has developed a rigorous and elegantly symmetric counterpart to the classical statistics of the familiar linear-normal regression setup.[4] The Bayesian dual counterpart to classical statistics works with a normal-gamma family of conjugate distributions. For reasons that will later become apparent, we work here with a three-parameter generalization of the two-parameter gamma distribution, which forms a *normal-truncated-gamma* family of conjugate

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[4] Among other places, clear expositions of Bayesian-classical duality are contained in DeGroot (1970), Zellner (1971), Leamer (1978), Hamilton (1995), and Poirier (1995).