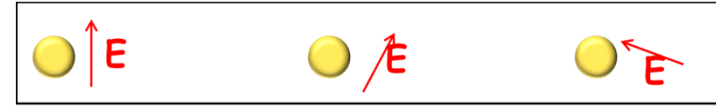


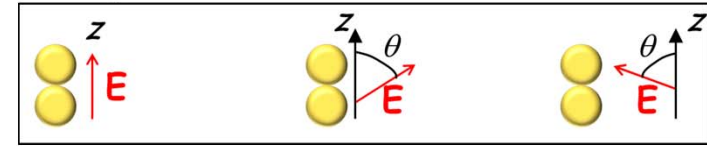
Theory of optical trapping of ultracold particles

Spherical atoms 1S or 2S : isotropic trapping



$$U(\mathbf{r}; \omega) = -\frac{1}{2\epsilon_0 c} \text{Re}[\alpha_{\text{scal}}(\omega)] \times I(\mathbf{r}) \quad \Gamma(\mathbf{r}; \omega) = \frac{1}{\hbar\epsilon_0 c} \text{Im}[\alpha_{\text{scal}}(\omega)] \times I(\mathbf{r})$$

Non-spherical atoms or molecules: **anisotropic** trapping



$$U_{M_J}^{\text{ell}}(\mathbf{r}; \theta_p, \theta_k, \mathcal{A}; \omega) = -\frac{1}{2\epsilon_0 c} I(\mathbf{r}) \left\{ \text{Re}[\alpha_{\text{scal}}(\omega)] + \mathcal{A} \cos \theta_k \frac{M_J}{2J} \text{Re}[\alpha_{\text{vect}}(\omega)] + \frac{3M_J^2 - J(J+1)}{J(2J+1)} \times \frac{3 \cos^2 \theta_p - 1}{2} \text{Re}[\alpha_{\text{tens}}(\omega)] \right\}$$

$$\Gamma_{M_J}^{\text{ell}}(\mathbf{r}; \theta_p, \theta_k, \mathcal{A}; \omega) = \frac{1}{\hbar\epsilon_0 c} I(\mathbf{r}) \left\{ \text{Im}[\alpha_{\text{scal}}(\omega)] + \mathcal{A} \cos \theta_k \frac{M_J}{2J} \text{Im}[\alpha_{\text{vect}}(\omega)] + \frac{3M_J^2 - J(J+1)}{J(2J+1)} \times \frac{3 \cos^2 \theta_p - 1}{2} \text{Im}[\alpha_{\text{tens}}(\omega)] \right\}$$

$$\alpha_{\text{scal}}(\omega) = \frac{1}{3(2J+1)} \sum_{\beta' J'} \left(\frac{\langle \beta' J' \| d \| \beta J \rangle^2}{E_{\beta' J'} - E_{\beta J} - i \frac{\hbar\gamma_{\beta' J'}}{2} - \hbar\omega} + \frac{\langle \beta' J' \| d \| \beta J \rangle^2}{E_{\beta' J'} - E_{\beta J} - i \frac{\hbar\gamma_{\beta' J'}}{2} + \hbar\omega} \right)$$

$$\alpha_{\text{vect}}(\omega) = \sqrt{\frac{6J}{(J+1)(2J+1)}} \sum_{\beta' J'} (-1)^{J+J'} \begin{Bmatrix} 1 & 1 & 1 \\ J & J & J' \end{Bmatrix} \times \left(\frac{\langle \beta' J' \| d \| \beta J \rangle^2}{E_{\beta' J'} - E_{\beta J} - i \frac{\hbar\gamma_{\beta' J'}}{2} - \hbar\omega} - \frac{\langle \beta' J' \| d \| \beta J \rangle^2}{E_{\beta' J'} - E_{\beta J} - i \frac{\hbar\gamma_{\beta' J'}}{2} + \hbar\omega} \right)$$

$$\alpha_{\text{tens}}(\omega) = 2 \sqrt{\frac{5J(2J-1)}{(J+1)(2J+1)(2J+3)}} \sum_{\beta' J'} (-1)^{J+J'} \begin{Bmatrix} 1 & 1 & 2 \\ J & J & J' \end{Bmatrix} \times \left(\frac{\langle \beta' J' \| d \| \beta J \rangle^2}{E_{\beta' J'} - E_{\beta J} - i \frac{\hbar\gamma_{\beta' J'}}{2} - \hbar\omega} + \frac{\langle \beta' J' \| d \| \beta J \rangle^2}{E_{\beta' J'} - E_{\beta J} - i \frac{\hbar\gamma_{\beta' J'}}{2} + \hbar\omega} \right)$$

Example:
Lanthanide atoms have strong magnetic moment

Er (3H_6): $7\mu_B$.
Dy (5I_8): $10\mu_B$.