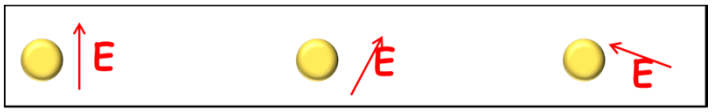
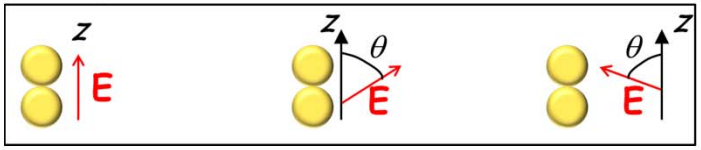
Theory of optical trapping of ultracold particles

Spherical atoms ^{1}S or ^{2}S: isotropic trapping



$$U(\mathbf{r};\omega) = -\frac{1}{2\epsilon_0 c}\mathrm{Re}[\alpha_{\mathrm{scal}}(\omega)] \times I(\mathbf{r}) \qquad \Gamma(\mathbf{r};\omega) = \frac{1}{\hbar\epsilon_0 c}\mathrm{Im}[\alpha_{\mathrm{scal}}(\omega)] \times I(\mathbf{r})$$

Non-spherical atoms or molecules: **anisotropic** trapping



$$U^{\mathrm{ell}}_{M_J}(\mathbf{r};\theta_p,\theta_k,\mathcal{A};\omega) = -\frac{1}{2\epsilon_0 c}I(\mathbf{r})\left\{\mathrm{Re}[\alpha_{\mathrm{scal}}(\omega)] + \mathcal{A}\cos\theta_k\frac{M_J}{2J}\mathrm{Re}[\alpha_{\mathrm{vect}}(\omega)] + \frac{3M_J^2 - J(J+1)}{J(2J+1)} \times \frac{3\cos^2\theta_p - 1}{2}\mathrm{Re}[\alpha_{\mathrm{tens}}(\omega)]\right\}$$

$$\Gamma^{\mathrm{ell}}_{M_J}(\mathbf{r};\theta_p,\theta_k,\mathcal{A};\omega) = \frac{1}{\hbar\epsilon_0 c}I(\mathbf{r})\left\{\mathrm{Im}[\alpha_{\mathrm{scal}}(\omega)] + \mathcal{A}\cos\theta_k\frac{M_J}{2J}\mathrm{Im}[\alpha_{\mathrm{vect}}(\omega)] + \frac{3M_J^2 - J(J+1)}{J(2J+1)} \times \frac{3\cos^2\theta_p - 1}{2}\mathrm{Im}[\alpha_{\mathrm{tens}}(\omega)]\right\}$$

$$\alpha_{\mathrm{scal}}(\omega) = \frac{1}{3(2J+1)}\sum_{\beta' J'}\left(\frac{\langle\beta' J'\|\,\mathrm{d}\,\|\beta J\rangle^2}{E_{\beta' J'} - E_{\beta J} - i\frac{\hbar\gamma_{\beta' J'}}{2} - \hbar\omega} + \frac{\langle\beta' J'\|\,\mathrm{d}\,\|\beta J\rangle^2}{E_{\beta' J'} - E_{\beta J} - i\frac{\hbar\gamma_{\beta' J'}}{2} + \hbar\omega}\right)$$

$$\alpha_{\mathrm{vect}}(\omega) = \sqrt{\frac{6J}{(J+1)(2J+1)}}\sum_{\beta' J'}(-1)^{J+J'}\left\{\begin{array}{ccc} 1 & 1 & 1 \\ J & J & J' \end{array}\right\} \times \left(\frac{\langle\beta' J'\|\,\mathrm{d}\,\|\beta J\rangle^2}{E_{\beta' J'} - E_{\beta J} - i\frac{\hbar\gamma_{\beta' J'}}{2} - \hbar\omega} - \frac{\langle\beta' J'\|\,\mathrm{d}\,\|\beta J\rangle^2}{E_{\beta' J'} - E_{\beta J} - i\frac{\hbar\gamma_{\beta' J'}}{2} + \hbar\omega}\right)$$

$$\alpha_{\mathrm{tens}}(\omega) = 2\sqrt{\frac{5J(2J-1)}{(J+1)(2J+1)(2J+3)}}\sum_{\beta' J'}(-1)^{J+J'}\left\{\begin{array}{ccc} 1 & 1 & 2 \\ J & J & J' \end{array}\right\} \times \left(\frac{\langle\beta' J'\|\,\mathrm{d}\,\|\beta J\rangle^2}{E_{\beta' J'} - E_{\beta J} - i\frac{\hbar\gamma_{\beta' J'}}{2} - \hbar\omega} + \frac{\langle\beta' J'\|\,\mathrm{d}\,\|\beta J\rangle^2}{E_{\beta' J'} - E_{\beta J} - i\frac{\hbar\gamma_{\beta' J'}}{2} + \hbar\omega}\right)$$

Example:
Lanthanide atoms have strong magnetic moment

Er ($^3H_6$): $7\mu_B$.
Dy($^5I_8$): $10\mu_B$.

Théomol