

$\hat{\psi}_k^j(+)$ , that sample the a posteriori error vectors  $\mathbf{E}_k(+)$  with variances  $\Pi_k(+)$ ; the ensemble covariance matrix from  $\hat{\psi}_k(+)$  then tends to  $\mathbf{P}_k^p(+)$  for  $q \rightarrow \infty$  (Eq. (A25)). The data residuals of all  $\hat{\psi}_k^j(+)$ 's are here also constrained to be in close accord with the measurement errors: the  $\hat{\psi}_k^j(+)$  which have residuals of horizontal-averaged variance larger than twice the local data error variance are rejected (Eq. (A25)). While ensemble forecasts are computed in parallel (Eq. (A26)), the SVD of the current error forecast matrix,  $M_{k+1}(-) = [\hat{\psi}_{k+1}^j(-) - \hat{\psi}_{k+1}(-)] \in \mathbb{R}^{n \times q}$ , is evaluated (Eq. (A27)). Integrations are stopped when the dominant SVD of these error samples stabilizes. This is here measured by the similarity coefficient  $\rho$  defined by Eq. (A28), where  $(\mathbf{E}, \Pi)$  of rank  $p$  and  $(\tilde{\mathbf{E}}, \tilde{\Pi})$  of rank  $\tilde{p} \geq p$  define “previous” and “new” estimates of the ES forecast, respectively. When  $\rho$  is close enough to one, the resulting  $(\tilde{\mathbf{E}}, \tilde{\Pi})$  determine the ES forecast for  $t_{k+1}$ ,  $(\Pi_{k+1}(-), \mathbf{E}_{k+1}(-))$ , to be used as in Section A.2. The dimensions of the ensemble ( $q$ ) and ES ( $p$ ) hence vary with time, in accord with data and dynamics. In passing, the nonlinearities ensure that each new linearly perturbed state  $\hat{\psi}_k^j(+)$  (Eq. (A25)) has the potential to add new value to the ES forecast.

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