

days later, he rushed into the graduate student's office with an ear-to-ear grin. By analyzing the at-sea field forecasts, he had also identified in real-time the ABV, IBV and MCC as features of dominant variability. He did not need the covariance eigendecompositions. For those of us who have had the chance of being at-sea and in real-time oceanography pressures, one can only be fascinated. It is a pleasure to acknowledge the continued stimulation, interest and encouragements of such a prodigious scientist of the real ocean. Several people have contributed to this work. I benefited greatly from the members of the Harvard oceanography group, past and present. In particular, I am thankful to Dr. Carlos J. Lozano for some comments. I am grateful to two anonymous referees for their helpful reviews. I thank Mr. Michael Landes and Mr. Todd Alcock for preparing some figures for this manuscript. This study was supported in part by the Office of Naval Research under grants N00014-95-1-0371 and N00014-97-1-0239 to Harvard University.

### Appendix A. Error subspace statistical estimation scheme employed

The main notation used and the estimation scheme employed are summarized. The intent is simply to provide an helpful and concise overview. For more on such methodologies, we refer to Lermusiaux (1997, 1998), Lermusiaux et al. (1998), and Lermusiaux and Robinson (1998). Related ensemble techniques for nonlinear data assimilation are addressed in (Evensen, 1994; Burgers et al., 1998; Miller et al., 1998). The so-called reduced state-space Kalman filters are discussed in a geophysical context for example by Cane et al. (1996) and Cohn and Todling (1996). The aim here is to reduce the error statistics in a fashion consistent with the assimilation criterion used. The present framework is that of a continuous-discrete estimation (Jazwinski, 1970). The gridded values of the PE fields,  $\hat{u}$ ,  $\hat{v}$ ,  $T$ ,  $S$  and  $\psi$  are combined into the state vector  $\psi = (\hat{u}, \hat{v}, \mathbf{T}, \mathbf{S}, \mathbf{p})^T \in \mathbb{R}^n$ . For the internal velocities  $\hat{u}$ ,  $\hat{v}$  the convention of Cox (1984) is kept; in all other cases,  $(\hat{\cdot})$  is the "estimate" operator (Gelb, 1974). Model errors are assumed null. The dynamical evolution of the ocean state  $\psi$  is described by,

$$d\psi = \mathbf{f}(\psi, t)dt, \quad (\text{A1})$$

where  $\mathbf{f}(\cdot, t)$  is the nonlinear PE operator, including boundary conditions and forcings. Data at time  $t_k$  are stored in  $\mathbf{d}_k \in \mathbb{R}^m$ . The measurement model associated with Eq. (A1) is

$$\mathbf{d}_k = \mathbf{C}_k \psi_k + \mathbf{v}_k. \quad (\text{A2})$$

The  $\mathbf{v}_k \in \mathbb{R}^m$  are random processes, assumed of zero statistical mean and of covariance matrix  $\mathbf{R}_k$ , with  $\varepsilon\{\mathbf{v}_k \mathbf{v}_j^T\} = 0$  for  $k \neq j$ . The state error covariance matrix at  $t_k$  is defined by  $\mathbf{P}_k \doteq \varepsilon\{(\hat{\psi}_k - \psi_k)(\hat{\psi}_k - \psi_k)^T\} \in \mathbb{R}^{n \times n}$ . The notation  $\varepsilon\{\mathbf{g}\}$  refers to the statistical mean of a given state space functional  $\mathbf{g}$ . At times  $t_k$ , to refer to quantities before and after the assimilation, the adjectives a priori (–) and a posteriori (+) are used, as in Gelb (1974). When the index  $k$  can be omitted, the (–) and (+) besides singular vectors are simplified to subscripts.