2.4.1.1. Measurement model parameters. The measurement model parameters (Section A.2) are the operator **C** and error covariance matrix **R**. Presently, the T/S profiles are first mapped onto the PE model levels by vertical linear interpolation. The matrix **C** then bilinearly interpolates the tracer forecast onto these profile-level intersections, or so-called data-points. The measurement error co-variance matrix $\mathbf{R} = [r_{ij}]$ is assumed diagonal, of elements r_{ii} a function of the internal dynamics, with a time and depth dependent amplitude:

$$r_{ii}^{\upsilon}(z,t) = \sigma^{\upsilon^2}(t) + \epsilon^{\upsilon}(t)\sigma_r^{\upsilon^2}(z,t), \tag{1}$$

where the superscript υ specifies the measured variable, t is the time and z the vertical coordinate. In Eq. (1), r_{ii}^{υ} is assumed to consist of two uncorrelated components: σ^{υ^2}, which accounts for the precision of the sensor utilized, human errors and forward interpolation errors; and the environmental noise, here estimated by $\epsilon^{\upsilon}\,\sigma_{\mathrm{r}}^{\upsilon^2}$, where $\sigma_{\mathrm{r}}^{\upsilon^2}$ is the internal, residual variability variance of the field υ and ϵ^{υ} is a factor to scale noise variances. The present values of σ^{υ} were estimated to 0.03°C for T profiles and to 0.01 PSU for S profiles. The comprehensive computation of the environmental noise is challenging. It often involves the estimation of the internal wave field and sub-mesoscale phenomena (Turner, 1981; Munk, 1981) from larger scale data (e.g. Flierl and Robinson, 1977). Several approaches for estimating $\sigma_{\mathrm{r}}^{\upsilon^2}(z,t)$ of the 20–35 km resolution data (Fig. 4b–d) were tested, but the model chosen is still a simple one. For initial conditions, $\sigma_{\mathrm{r}}^{\upsilon^2}(z,0)$ at a given depth is set to the horizontal average of the square of the differences, $\mathbf{d}_{\mathrm{his}} - C\hat{\psi}_0$, between the historical data ($\mathbf{d}^{\mathrm{his}}$, Fig. 4a) and initial conditions ($\hat{\psi}_0$, Fig. 5). The evolution of $\sigma_{\mathrm{r}}^{\upsilon^2}(z,t)$ is estimated as follows. At melding time, $\sigma_{\mathrm{r}}^{\upsilon^2}$ at a given depth is set to the horizontal average of the variance at data-points of the ensemble forecasts with respect to the data to be assimilated. Simply assuming proportionality with this variability, $\epsilon^{\upsilon}\,\sigma_{\mathrm{r}}^{\upsilon^2}(z,t)$ is then an approximate forecast of the environmental noise. The values of ϵ^{υ} were set to 0.25 for $\upsilon = T$ and to 0.3 for $\upsilon = S$. This quantitative scheme for forecasting the environmental noise led to robust assimilations. From our sensitivity studies, models like Eq. (1) play an important role. For example, if the estimated (environmental) data error variance is too small or of erroneous distribution, scales that are not of interest can spoil the state estimate. Further theoretical and observational research is required for more advanced formulations.

2.4.1.2. Adaptive ES learning parameters. The present scheme is adaptive (Section A.3): the possibly significant a posteriori data residuals are employed to correct (learn) the ES. These residuals are first objectively analyzed and then used to update the a posteriori ES. The main parameters in the ES learning pertain to the mapping of the residuals. This mapping is carried out here by a multivariate ESSE analysis (Lermusiaux et al., 1998), assuming that the background error covariance function is separable in the horizontal and vertical. From Kronecker product properties (Graham, 1981), the dominant eigendecomposition of the background error covariance matrix is then obtained from the these of the horizontal and vertical error covariance matrices. In the horizontal, the covariance function is assumed isotropic mesoscale (Section 2.3), of decorrelation scales set to 25 km, and zero-crossings to 50 km. In the vertical, the eigendecomposition is estimated from the vertical EOFs of the residuals.