

By replacing assumptions (2) through (5) in Theorem 4.2 by

(2') for each fixed $\omega \in \Omega$ the function $P_i(\omega, \cdot)$ is l.s.c.,

(3') $\text{co } F_i$ is lower measurable,

(4') for each measurable function $x: \Omega \rightarrow X$ we have $x_i(\omega) \notin \text{co } F_i(\omega, x(\omega))$ for almost all $\omega \in \Omega$, and

(5') P_i is integrably bounded and has a measurable graph,

and invoking [34, Theorem 3.2] (which asserts that the integral of a l.s.c. correspondence which is integrably bounded and has a measurable graph is also l.s.c.), we can guarantee that for each fixed $\omega \in \Omega$ the function $F_i(\omega, \cdot)$ is l.s.c. Therefore, by appealing to Theorem 3.2 one can prove the existence of a Bayesian equilibrium for this more general form of a Bayesian game.

(B) In Section 5 we remarked that if the dimensionality of the strategy space is infinite, then $\int \overline{X^\epsilon} = \int X = \int \overline{\text{co}}(X^\epsilon)$ and consequently only an approximate pure strategy equilibrium could be found. However, by assuming that there are "many more players than strategies," i.e., if the "dimension" of the measure space of players is larger than the "dimension" of the strategy space, one can remove the norm closure and obtain $\int X^\epsilon = \int X = \int \overline{\text{co}}(X^\epsilon)$. Hence an exact pure strategy equilibrium can be obtained. Of course, the concept of dimension has to be given a rigorous formulation. See [30] for a further discussion.

References

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