We would like to note that in our framework the expected utility is required to be only quasiconcave in each player's own strategy. Moreover, our expected utility is random, i.e., depends on the state of the world. The latter is quite important since with random expected payoffs the Fan–Glicksberg result is not directly applicable and the use of measurable selection theorems seems to be needed.

Although it is not obvious how one from the approach of Milgrom–Weber and Balder can obtain versions of our Theorems 4.2 and 5.1, it is very clear that these theorems are not subsumed by any of their results. In particular, no assumption of equicontinuity of payoffs is needed and the set of players in Theorem 4.2 is not necessarily finite. It may be instructive to note that our approach, i.e., working with strategies which are measurable functions, seems to be quite natural to analyze economies with incomplete information as recently by T. Palfrey and S. Srivastava [23, 24] and A. Postlewaite and D. Schmeidler [26] or uncertainty in market games examined in J. Peck and K. Shell [25]. In fact, our approach as well as Theorems 4.2 and 5.1 have been motivated from the work of the above authors.

Finally, we would like to note that A. Mas-Colell [18], viewing a game as a probability measure on the space of utility functions, has proved Nash equilibrium existence theorems. He also indicates that his existence results may be useful to obtain results for games with incomplete information.

7. Concluding remarks

We now show how a version of Theorem 3.4 can be easily obtained by combining the deterministic equilibrium result of N. C. Yannelis and N. D. Prabhakar [32] with the Aumann Measurable Selection Theorem.

Theorem 7.1 *The conclusion of Theorem 3.4 remains true if one replaces assumptions* (2) *and* (4) *by:*

(2′) $\operatorname{co} P_i$ *is lower measurable, i.e., for every open set* V *in* X_i *the set*

$$\{(\omega, x) : \operatorname{co} P_i(\omega, x) \cap V \neq \varnothing\}$$

belongs to $\Sigma \otimes \beta(X)$*; and*

(4′) *For each* $\omega \in \Omega$ *the function* $P_i(\omega, \cdot)$ *has open lower sections, i.e., for each* $\omega \in \Omega$ *and for each* $y_i \in X_i$ *the set* $P^{-1}(\omega, y_i) = \{x \in X : y_i \in P_i(\omega, x)\}$ *is open in* X.

Proof: For each $i \in I$ define $\varphi_i : \Omega \times X \to 2^{X_i}$ by $\varphi_i(\omega, x) = \operatorname{co} P_i(\omega, x)$. By assumption (a) each function φ_i is lower measurable. Define the correspondence $F : \Omega \times X \to 2^X$ by $F(\omega, x) = \Pi_{i \in I} \varphi_i(\omega, x)$. By virtue of Theorem 2.6 the function F is lower measurable. Define the correspondence $\Gamma : \Omega \to 2^X$ by

$$\Gamma(\omega) = \{x \in X : F(\omega, x) = \varnothing\}.$$