compact). Hence, the correspondence F satisfies all the conditions of the Fan–Glicksberg Fixed Point Theorem. Consequently, there exists some $x^* \in L_X$ such that $x^* \in F(x^*)$. Now it is a routine matter to verify that x^* is a Bayesian equilibrium for $\mathcal{G} = \{(X_i, h_i, S, q_i) : i = 1, 2, \ldots, n\}$. This completes the proof of the theorem. ∎

Remarks. The proof of Theorem 5.1 remains unchanged if (2) is replaced by:

(2′) $X_i : \Omega \to 2^Y$ is a nonempty, weakly compact and convex valued correspondence having a measurable graph.

We now indicate how one can prove the existence of a pure strategy Bayesian equilibrium. Denote by X_i^e the set of all extreme points of X_i. A *pure strategy Bayesian equilibrium* for $\mathcal{G} = \{(X_i, h_i, S_i, q_i) : i \in I\}$ is some element x^* in the set

$$\prod_{i=1}^{n} \int X_i^e(\omega)\, d\mu(\omega) =$$

$$\prod_{i=1}^{n} \Big\{ \int z_i(\omega)\, d\mu(\omega) : z_i(\cdot) \text{ is } S_i\text{-measurable and } z_i(\omega) \in X_i^e(\omega)\ \mu\text{-a.e.} \Big\}$$

such that for all i, we have

$$v_i(\omega, x^*) = \max_{y_i \in \int X_i^e} v_i(\omega, x_1^*, \ldots, x_{i-1}^*, y_i, x_{i+1}^*, \ldots, x_n^*)$$

for almost all $\omega \in \Omega$ (where v_i is defined as in (5.1)).

For the next result, we will assume that S_i is a σ-subalgebra of Σ.

Theorem 5.2 *Let $\mathcal{G} = \{(X_i, h_i, S_i, q_i) : i = 1, 2, \ldots, n\}$ be a Bayesian game satisfying assumptions* (1), (4), (6), *and* (7) *of Theorem 5.1 in addition to the following conditions:*

1. (Ω, S_i, μ) *is an atomless measure space,*

2. $X_i : \Omega \to 2^{R^\ell}$ *is a nonempty, closed, convex and integrably bounded correspondence with a measurable graph and for each $\omega \in \Omega$ the function $h_i(\omega, \cdot)$ is linear and continuous on X.*

Then there exists a pure strategy Bayesian equilibrium for $\mathcal{G}$.

Proof: First note that since for each fixed $\omega \in \Omega$ the function $h_i(\omega, \cdot)$ is linear and continuous on X the domain of v_i is now $\Omega \times \Pi_{i=1}^n \int X_i$. Let $\int X = \Pi_{i=1}^n \int X_i$. The set $\int X$ will turn out to be equal to $\int X^e = \Pi_{i=1}^n \int X_i^e$ as we shall show below.