The proof of this claim is similar to that of Theorem 3.1 in [36] but we provide an outline for the sake of completeness. First note that since (Ω, Σ, μ) is separable and Y is separable, $L_1(\mu, Y)$ is a separable Banach space. Since by assumption each X_i is nonempty, convex and weakly compact, it follows from Diestel's Theorem that each L_{X_i} is a weakly compact subset of $L_1(\mu, Y)$. Obviously, each L_{X_i} is convex since each X_i is convex and by virtue of the Aumann Measurable Selection Theorem, we can conclude that each L_{X_i} is also nonempty. Furthermore, since each L_{X_i} is a weakly compact subset of the separable Banach space $L_1(\mu, Y)$, it is also metrizable; see [1, Theorem 10.11, p. 154]. Clearly, $L_X = \Pi_{i \in I} L_{X_i}$ is nonempty, convex, weakly compact and metrizable as well.

II. *The function $v_i(\omega, \cdot)$ is weakly continuous for each $\omega \in \Omega$.*

Fix $i \in I = \{1, 2, \ldots, n\}$, $\omega \in \Omega$ and $E_i(\omega) \in S_i$. Let $\{x_n\}$ be a sequence of L_X converging weakly[5] to $x \in L_X$, i.e., the sequence $\{x_n^i : n = 1, 2, \ldots\}$ of L_{X_i} converges weakly to $x^i \in L_{X_i}$ for each $i \in I$. We must show that the sequence $\{x_n^i \chi_{E_i(\omega)}\}$ converges pointwise in the weak topology of X_i to $x^i \chi_{E(\omega)}$ for each i. Then in view of (3) and (6) the result will follow from the Lebesgue Dominated Convergence Theorem.

Now if $S_i = \{E_i^1, E_i^2, \ldots\}$ is a partition of player i, then the fact that x_n^i and x^i belong to L_{X_i} implies that

$$x_n^i = \sum_{k=1}^{\infty} x_n^{i,k} \chi_{E_i^k} \quad \text{and} \quad x^i = \sum_{k=1}^{\infty} x^{i,k} \chi_{E_i^k},$$

with $x_n^{i,k}$, $x^{i,k} \in X_i$, and therefore we can conclude that

$$x_n^i \chi_{E_i(\omega)} = \sum_{k=1}^{\infty} x_n^{i,k} \chi_{E_i^k \cap E_i(\omega)}$$

converges weakly to $x^i \chi_{E_i(\omega)} = \sum_{k=1}^{\infty} x^{i,k} \chi_{E_i^k \cap E_i(\omega)}$.

III. *Each correspondence $\varphi_i \colon L_{\hat{X}_i} \to 2^{L_{x_i}}$ is nonempty, convex valued and weakly u.s.c.*

It follows from assumption (5) that for each $\omega \in \Omega$ and for each $\hat{x} \in L_{\hat{X}_i}$ that $v_i(\omega, x_i, \hat{x}_i)$ is a concave function of x_i on L_{X_i}, and therefore we can conclude that φ_i is convex valued. By virtue of Berge's Maximum Theorem (see for instance [7, Theorem 12.1]), we see that φ_i is weakly u.s.c. Finally, an appeal to the Weierstrass' Theorem guarantees that φ_i is also a nonempty valued correspondence.

Now since each φ_i is nonempty, closed, convex valued and weakly u.s.c., it follows from Theorem 2.2 that likewise is $F \colon L_X \to 2^{L_X}$ (and L_X is weakly

[5]Let $\{f_n\}$ be a sequence in $L_1(\mu, Y)$. Then $\{f_n\}$ converges weakly to f if and only if $\langle f_n, p \rangle$ (the value of f_n at p) converges to $\langle f, p \rangle$ for any $p \in L_\infty(\mu, Y^*)$ (recall that Y^* has the RNP). The latter is equivalent to saying that $\langle f_n \chi_A, p \rangle = \langle f_n, \chi_A p \rangle$ converges to $\langle f, \chi_A p \rangle = \langle f_n \chi_A, p \rangle$ for each $p \in L_\infty(\mu, Y^*)$ and each $A \in \Sigma$. Each condition above implies that $\langle f_n \chi_A, x^* \rangle = \langle f_n, \chi_A x^* \rangle$ converges to $\langle f \chi_A, x^* \rangle = \langle f, \chi_A x^* \rangle$ for each $x^* \in Y^*$ and each $A \in \Sigma$.