Let $L_X = \Pi_{i\in I} L_{X_i}$. Denote by $E_i(\omega)$ the event in S_i containing the true state of nature $\omega \in \Omega$ and suppose that $q_i(E_i(\omega)) > 0$ for all $i \in I$. Given $E_i(\omega)$ in S_i define the conditional expected utility function $v_i: \Omega \times L_X \to R$ of player i by

$$v_i(\omega, x) = \int_{E_i(\omega)} q_i\big(t|E_i(\omega)\big) h_i\big(t, x(t)\big)\, d\mu(t). \tag{5.1}$$

A *Bayesian equilibrium* for $\mathcal{G} = \{(X_i, h_i, S, q_i) : i \in I\}$ is an element $x^* \in L_X$ such that

$$v_i(\omega, x^*) = \max_{y_i \in L_{X_i}} v_i\big(\omega, x_1^*(\omega), \ldots, x_{i-1}^*(\omega), y_i, x_{i+1}^*(\omega), \ldots\big),$$

where v_i is given by formula (5.1).

We now state the following result.

Theorem 5.1 *Let $\mathcal{G} = \{(X_i, h_i, S, q_i) : i = 1, 2, \ldots, n\}$ be a Bayesian game satisfying the properties:*

1. *the measure space (Ω, Σ, μ) is finite, separable and complete,*
2. *each X_i is a nonempty, convex, and weakly compact subset of a separable Banach space Y whose dual Y^* has the RNP,*
3. *each function $h_i(\omega, \cdot)$ is weakly continuous,*
4. *each function $h_i(\cdot, x)$ is measurable,*
5. *for each $\omega \in \Omega$ and each $\hat{x} \in \hat{X}$ the function $h_i(\omega, x_i, \hat{x}_i)$ is concave in x_i, and*
6. *each h_i is integrably bounded.*

Then $\mathcal{G}$ has a Bayesian equilibrium.

Proof: For each $i \in I$ define the correspondence $\varphi_i: L_{\hat{X}_i} \to 2^{L_{X_i}}$ by

$$\varphi(\hat{x}_i) = \Big\{y_i \in L_{X_i} : v_i(\omega, y_i, \hat{x}_i) = \max_{x_i \in L_{X_i}} v_i(\omega, x_i, \hat{x}_i) \text{ for almost all } \omega \in \Omega\Big\}.$$

Also, define the correspondence $F: L_X \to 2^{L_X}$ by

$$F(x) = \prod_{i\in I} \varphi_i(\hat{x}_i).$$

We shall show that the correspondence F satisfies the hypotheses of the Fan–Glicksberg Fixed Point Theorem (see for instance [12]). It can be easily seen that a fixed point of the correspondence F is by construction a Bayesian equilibrium for the game $\mathcal{G}$. We shall complete the proof by several steps.

I. *L_X is nonempty, convex, weakly compact and metrizable.*