2. *$h_i: \Omega \times X \to R$ (where $X = \Pi_{i \in I} X_i$) is the random payoff function of player i,*

3. *S_i is a measurable partition of (Ω, Σ) denoting the (private) information available to player i, and*

4. *$q_i: \Omega \to (0, \infty)$ is the prior probability density of player i, i.e., q_i is a measurable function having the property that $\int_\Omega q_i(t)\, d\mu(t) = 1$.*

As in R. J. Aumann [3] or R. Myerson [21] it is assumed that the game $\mathcal{G} = \{(X_i, h_i, S_i, q_i) : i \in I\}$ is common knowledge, i.e., every player knows $\mathcal{G}$, every player knows that every player knows $\mathcal{G}$, every player knows that every player knows that every player knows $\mathcal{G}$, and so on.

We first consider the case where the information set of each player i is the same, i.e., $S_i = S$ for each $i \in I$. Denote by $E(\omega)$ the event in S which contains the realized state of nature $\omega \in \Omega$, and suppose that $q_i(E(\omega)) > 0$ for all $i \in I$. Given $E(\omega)$ in S the conditional expected utility of player i is the function $v_i: \Omega \times X \to R$ defined by

$$v_i(\omega, x) = \int_{E(\omega)} q_i\big(t|E(\omega)\big) h_i(t, x)\, d\mu(t), \tag{4.1}$$

where

$$q_i\big(t|E(\omega)\big) = \begin{cases} 0, & \text{if } t \notin E(\omega); \\ \dfrac{q_i(t)}{\int_{E(\omega)} q_i(s)\, d\mu(s)}, & \text{if } t \in E(\omega)\,. \end{cases}$$

A *Bayesian equilibrium* for a Bayesian game

$$\mathcal{G} = \{(X_i, h_i, S_i, q_i) : i \in I\}$$

is a function $x^*: \Omega \to X$ such that each $x_i^*(\cdot)$ is S-measurable and for each $i \in I$ we have

$$v_i\big(\omega, x^*(\omega)\big) = \max_{y_i \in X_i} v_i\big(\omega, x_1^*(\omega), \ldots, x_{i-1}^*(\omega), y_i, x_{i+1}^*(\omega), \ldots\big)$$

for almost all $\omega \in \Omega$, where v_i is given by (4.1).

We are now ready to state our fist Bayesian equilibrium existence theorem.

Theorem 4.2 *Let $\mathcal{G} = \{(X_i, h_i, S_i, q_i) : i \in I\}$ be a Bayesian game satisfying for each $i \in I$ the following properties.*

1. *each X_i is a nonempty, compact and convex subset of a separable Banach space Y,*

2. *for each fixed $\omega \in \Omega$ the function $h_i(\omega, \cdot)$ is continuous,*

3. *for each fixed $x \in X$ the function $h_i(\cdot, x)$ is measurable,*

4. *for each $\omega \in \Omega$ and each $\hat{x} \in \hat{X}_i (= \Pi_{j \neq i} X_j)$ the function $h_i(\omega, x_i, \hat{x}_i)$ is concave in x_i, and*