The following corollary of Theorem 3.4 extends Corollary 3.3 to strategy sets which may be subsets of arbitrary separable Banach spaces. We thus have a random version of Nash's result [22, Theorem 1, p. 288] in separable Banach spaces. It should be noted that Corollary 3.5 below may be seen as a random generalization of the deterministic equilibrium existence results of K. Fan [13, Theorem 4, p. 192] and F. E. Browder [8, Theorem 14, p. 277], but only if the underlying strategy space is separable. Note the latter assumption is needed in order to make the Aumann Measurable Selection Theorem applicable. It is worth noting that Fan and Browder allow only for a finite number of players whereas in our setting the set of players may be countable.

Corollary 3.5 *Replace assumption* (1) *in Corollary 3.3 by*

(1′) X_i *is a nonempty, compact, and convex subset of a separable Banach space.*

Then the conclusion of Corollary 3.3 remains true.

Proof: The proof is identical with that of Corollary 3.3 taking into account that one now has to use Lemma 2.13(a) to show that Q_i has a measurable graph, and appeal to Theorem 3.4 instead of Theorem 3.2. ∎

A couple of comments are in order. Notice that the continuity assumption (4) in Theorem 3.2 is weaker than the continuity assumption (4) of Theorem 3.4. The reason we need a weaker continuity assumption is that the proof of Theorem 3.2 makes use of Theorem 2.4 which is a Carathéodory selection result for a correspondence that is lower measurable in one variable and l.s.c. in the other. However, in the proof of Theorem 3.4 a different Carathéodory selection result is used (Theorem 2.5) which requires a stronger continuity assumption. Moreover, observe that Corollary 3.3 follows directly from Corollary 3.5. Nevertheless, we choose to state Corollary 2.11 since its proof by means of Theorem 3.2 is slightly different than the proof of Corollary 3.3 which follows from Theorem 3.4. Finally, it is important to note that the proofs of Theorems 3.2 and 3.4 do not use any deterministic equilibrium existence results. To the contrary, our arguments start from scratch and provide alternative ways to prove the equilibrium results of Nash, Fan, and Browder.[3]

4. Bayesian games and equilibria

We now turn to the problem of the existence of equilibrium points for Bayesian games. Again, let (Ω, Σ, μ) be a complete finite measure space. We still denote by I the set of players, where I can be finite or countable.

Definition 4.1 *A Bayesian game on the complete finite measure space* (Ω, Σ, μ) *is a set* $\mathcal{G} = \{(X_i, h_i, S_i, q_i) : i \in I\}$ *of quadruples such that*

1. *each* X_i *is the strategy set of player* i,

[3] An alternative proof of a version of Theorem 3.4 will be given in Section 7.