Notice that each player in the game described above is characterized by a strategy set and a random preference correspondence. We now follow the original formulation by J. Nash [22] (and its generalizations by Fan [12] and Browder [8] among others) where random preference correspondences are replaced by random payoff functions, i.e., real valued functions defined on $\Omega \times X$.

Let $\Gamma = \{(X_i, u_i) : i \in I\}$ be a *Nash-type random game*, i.e.,

1. X_i is the *strategy set* of player i, and
2. $u_i: \Omega \times X \to R$ (where $X = \Pi_{i \in I} X_i$) is the *random payoff function* of player i.

Let $\hat{X}_i = \Pi_{j \neq i} X_j$ and denote the elements of $\hat{X}_i$ by $\hat{x}_i$. A *random Nash equilibrium* for Γ is a measurable function $x^*: \Omega \to X$ such that for all i and almost all $\omega \in \Omega$ we have

$$u_i(\omega, x^*(\omega)) = \max_{y_i \in X_i} u_i(\omega, y_i, \hat{x}_i^*(\omega)).$$

We now state our first random equilibrium existence result.

Theorem 3.2 *Let $\mathcal{E} = \{(X_i, P_i) : i \in I\}$ be a random game satisfying for each i the following properties:*

1. *each X_i is a nonempty, compact, and convex subset of R^ℓ,*
2. *each $\operatorname{co} P_i$ is lower measurable, i.e., for every open subset V of X_i the set*

   $$\{(\omega, x) \in \Omega \times X : \operatorname{co} P_i(\omega, x) \cap V \neq \varnothing\}$$

   belongs to $\Sigma \otimes \beta(X)$,
3. *for every measurable function $x: \Omega \to X$ we have $x_i(\omega) \notin \operatorname{co} P_i(\omega, x(\omega))$ for almost all $\omega \in \Omega$, and*
4. *for each $\omega \in \Omega$ the set-valued function $P_i(\omega, \cdot)$ is l.s.c.*

Then there exists a random equilibrium for E.

Proof: For each $i \in I$ define the correspondence $\varphi_i: \Omega \times X \to 2^{X_i}$ by $\varphi_i(\omega, x) = \operatorname{co} P_i(\omega, x)$. Since by assumption (4) each $P_i(\omega, \cdot)$ is l.s.c., it follows from Theorem 2.1 that for each fixed $\omega \in \Omega$ the function $\varphi_i(\omega, \cdot)$ is l.s.c. Furthermore, by assumption (2), the function φ_i is lower measurable and clearly convex valued. Next, let

$$O_i = \{(\omega, x) \in \Omega \times X : \varphi(\omega, x) \neq \varnothing\},$$

and

$$O_i^\omega = \{x \in X : (\omega, x) \in O_i\} \quad \text{and} \quad O_i^x = \{\omega \in \Omega : (\omega, x) \in O_i\}.$$

From Theorem 2.4, it follows that there exists a Carathéodory selection f_i for φ_i. I.e., there exists a function $f_i: O_i \to X_i$ such that