variation there exists some $g \in L_1(\mu, Y)$ such that $G(E) = \int_E g(t)\,d\mu(t)$ for all $E \in \Sigma$. A Banach space Y has the *Radon–Nikodym Property* (RNP) if Y has the RNP with respect to every finite measure space. It is a standard result that if Y^* (the norm dual of Y) has the RNP, then $\big(L_1(\mu, Y)\big)^* = L_\infty(\mu, Y^*)$.

The correspondence $\varphi: T \to 2^Y$ is said to be *integrably bounded* if there exists a map $g \in L_1(\mu)$ such that $\sup\{\|x\| : x \in \varphi(t)\} \le g(t)$ holds for almost all $t \in T$. We denote by L_φ the set of all Y-valued Bochner integrable selections of $\varphi: T \to 2^Y$, i.e.,

$$L_\varphi = \{x \in L_1(\mu, Y) : x(t) \in \varphi(t) \text{ for almost all } t \text{ in } T\}.$$

Define the *integral* of the correspondence φ by

$$\int_T \varphi(t)\,d\mu(t) = \Big\{\int_T x(t)\,d\mu(t) : x \in L_\varphi\Big\}.$$

By Theorem 2.3 if T is a complete finite measure space, Y is a separable Banach space, and $\varphi: T \to 2^Y$ is a nonempty valued correspondence with a measurable graph (or equivalently φ is lower measurable and closed valued), then φ admits a measurable selection, i.e., there exists a measurable function $f: T \to Y$ such that $f(t) \in \varphi(t)$ for almost all $t \in T$. By virtue of this result (and provided that φ is integrably bounded), we can conclude that $L_\varphi \neq \emptyset$ and therefore $\int_T \varphi(t)\,d\mu(t) \neq \emptyset$.

Finally, we wish to note that *Diestel's Theorem* (see for instance [36, Theorem 3.1]) asserts that if F is an arbitrary nonempty, weakly compact, convex subset of a separable Banach space Y (or more generally if $F: T \to 2^Y$ is a nonempty, integrably bounded, and weakly compact convex valued correspondence) then L_F is a weakly compact subset of $L_1(\mu, F)$. With all these preliminary results out of the way, we can turn to our contributions.

3. Random games and equilibria

Let (Ω, Σ, μ) be a complete finite measure space. We interpret Ω as the states of nature of the world and assume that Ω is large enough to include all events that we consider to be interesting. As usual, Σ denotes the σ-algebra of events. Denote by I the set of players. The set I may be finite or countable.

Definition 3.1 *A* random game *is a set* $\mathcal{E} = \{(X_i, P_i) : i \in I\}$ *of ordered pairs, where*

1. X_i *is the* strategy set *of player* i*, and*
2. $P_i: \Omega \times X \to 2^{X_i}$ *(where* $X = \Pi_{i \in I} X_i$*) is the* random preference (or choice) correspondence *of player* i*.*

We read $y_i \in P_i(\omega, x)$ *as player* i *strictly prefers* y_i *to* x_i *at the state of nature* ω*, if the (given) components of the other players are fixed.*

A random equilibrium *for the game* E *is a measurable function* $x^*: \Omega \to X$ *such that for all* $i \in I$ *we have* $P_i\big(\omega, x^*(\omega)\big) = \emptyset$ *for almost all* $\omega \in \Omega$*.*