Proof: Define the correspondence $W: \Omega \times X \to 2^Y$ by

$$W(\omega, x) = \gamma(\omega, x) \cap \delta(\omega, x).$$

Since γ and δ are closed valued and lower measurable and at least one of them is compact valued, it follows from Theorem 2.7 that W is lower measurable. Define the correspondence $\varphi: \Omega \to 2^X$ by $\varphi(\omega) = \{x \in X : W(\omega, x) \neq \varnothing\}$. Observe that

$$\begin{aligned} G_\varphi &= \{(\omega, x) \in \Omega \times X : x \in \varphi(\omega)\} \\ &= \{(\omega, x) \in \Omega \times X : W(\omega, x) \neq \varnothing\} \\ &= \{(\omega, x) \in \Omega \times X : W(\omega, x) \cap Y \neq \varnothing\}, \end{aligned}$$

and the latter set belongs to $\Sigma \otimes \beta(X)$ since W is lower measurable. Consequently, $G_\varphi \in \Sigma \otimes \beta(X)$. It follows from Fan's Coincidence Theorem [12, Theorem 6, p. 238] that for each $\omega \in \Omega$ we have $\varphi(\omega) \neq \varnothing$. Thus, the correspondence $\varphi: \Omega \to 2^X$ satisfies all the conditions of Theorem 2.3 (the Aumann Measurable Selection Theorem) and consequently, there exists a measurable function $x^*: \Omega \to X$ such that $x^*(\omega) \in \varphi(\omega)$ for almost all ω in Ω, i.e., $\gamma(\omega, x^*(\omega)) \cap \delta(\omega, x^*(\omega)) \neq \varnothing$ for almost all ω in Ω. This completes the proof of the theorem. ∎

An immediate corollary of the above theorem is a random version of the Kakutani–Fan–Glicksberg Fixed Point Theorem [11, Theorem 1, p. 122].

Corollary 2.11 *Let X be a nonempty, compact and convex, subset of a locally convex separable and metrizable linear topological space Y and let (Ω, Σ, ν) be a complete finite measure space. Let $\gamma: \Omega \times X \to 2^X$ be a nonempty, closed, convex valued correspondence such that for each fixed $\omega \in \Omega$ the function γ is u.s.c. and γ is lower measurable. Then γ has a random fixed point, i.e., there exists a measurable function $x^*: \Omega \to X$ such that $x^*(\omega) \in \gamma(\omega, x^*(\omega))$ for almost all $\omega \in \Omega$.*

Proof: Define the correspondence $\delta: \Omega \times X \to 2^X$ by $\delta(\omega, x) = \{x\}$. Clearly, for each fixed $\omega \in \Omega$ the function $\delta(\omega, \cdot)$ is u.s.c. and δ is nonempty, convex, and compact valued and lower measurable. Let $x \in X$ and $\omega \in \Omega$. By choosing $u \in \gamma(\omega, x)$, $z = x \in \delta(\omega, x)$, and $\lambda \in (0, 1)$, the assumption (3) of Theorem 2.10 is satisfied. (Simply notice that $y = x + \lambda(u - z) = \lambda u + (1 - \lambda)x \in X$, since X is convex.) Hence, by the previous corollary, there exists a measurable function $x^*: \Omega \to X$ such that $\gamma(\omega, x^*(\omega)) \cap \delta(\omega, x^*(\omega)) \neq \varnothing$ for almost all $\omega \in \Omega$, i.e., $x^*(\omega) \in \gamma(\omega, x^*(\omega))$ for almost all $\omega \in \Omega$. ∎

Remarks. Theorem 2.10 and Corollary 2.11 remain true if we replace the assumption that (Ω, Σ, ν) is a complete finite (or σ-finite) measure space, by the fact that (Ω, Σ) is simply a measurable space. In this case one only needs to observe that in the proof of Theorem 2.10 for each fixed $\omega \in \Omega$ the function $W(\omega, \cdot)$ is u.s.c. (as it is the intersection of two u.s.c. correspondences) and therefore,