$f(x,\cdot)$ is continuous and for each fixed $z \in Z$ the function $f(\cdot,z)$ is measurable, then f is jointly measurable (where $\beta = \beta(Y)$ and $\Sigma = \beta(Z)$). It turns out that in several instances U is a proper subset of $X \times Z$, and this situation is more delicate. However, in this more delicate situation it can be shown that f is still jointly measurable. In particular, we have the following fact.

Theorem 2.8 *Let (Ω,α) be a measurable space, X be a separable metric space, Y a metric space and $U \subset \Omega \times X$ be such that:*

1. *For each $\omega \in \Omega$ the set $U^\omega = \{x \in X : (\omega,x) \in U\}$ is open in X, and*
2. *for each $x \in X$ the set $U^x = \{\omega \in \Omega : (\omega,x) \in U\}$ belongs to α.*

Let $f: U \to Y$ be a function such that for each $\omega \in \Omega$ the function $f(\omega,\cdot)$ is continuous on U^ω and for each $x \in X$ the function $f(\cdot,x)$ is measurable on U^x. Then f is jointly relatively measurable with respect to the σ-algebra $\alpha \otimes \beta(X)$, i.e., for every open subset V of Y we have

$$\{(\omega,x) \in U : f(\omega,x) \in V\} = U \cap A$$

for some $A \in \alpha \otimes \beta(X)$.

Proof: See [16, Lemma 4.12]. ∎

Theorem 2.9 *Let (Ω,α,μ) be a complete measure space and X be a complete separable metric space. If a set A belongs to $\alpha \otimes \beta(X)$, then its projection $\mathrm{proj}_\Omega(A)$ belongs to α.*

Proof: See [9, Theorem III.23, p. 75]. ∎

The next result is a random version of Fan's Coincidence Theorem. (See [12, Theorem 6, p. 238], and also [7, Theorem 17.1, p. 78].)

Theorem 2.10 *Let X be a nonempty, compact and convex subset of a locally convex separable and metrizable linear topological space Y and let (Ω,Σ,ν) be a complete finite measure space. Let $\gamma: \Omega \times X \to 2^Y$ and $\delta: \Omega \times X \to 2^Y$ be two nonempty, convex, closed and at least one of them compact valued correspondences such that:*

1. *γ and δ are both lower measurable,*
2. *for each $\omega \in \Omega$, the correspondences $\gamma(\omega,\cdot): X \to 2^Y$ and $\delta(\omega,\cdot): X \to 2^Y$ are both u.s.c., and*
3. *for every $(\omega,x) \in \Omega \times X$ there exist three points $y \in X$, $u \in \gamma(\omega,x)$, $z \in \delta(\omega,x)$ and a real number $\lambda > 0$ such that $y - x = \lambda(u - z)$.*

Then there exists a measurable function $x^*: \Omega \to X$ such that

$$\gamma(\omega, x^*(\omega)) \cap \delta(\omega, x^*(\omega)) \neq \varnothing$$

for almost all $\omega \in \Omega$.