Theorem 2.3 *Let (Ω, α, μ) be a complete finite measure space, X be a separable metric space and $\varphi: \Omega \to 2^Y$ be a nonempty valued correspondence having a measurable graph, i.e., $G_\varphi \in \alpha \otimes \beta(X)$. Then there exists a measurable selection for φ.*

Proof: See [9, Theorem III.22, p. 22] or [15, Theorem 5.2, p. 60]. ∎

Theorem 2.4 *Let (Ω, α, μ) be a complete finite measure space, X be a complete separable metric space and $\varphi: \Omega \times X \to 2^{R^\ell}$ be a convex (possibly empty) valued correspondence such that:*

1. *φ is lower measurable with respect to the σ-algebra $\alpha \otimes \beta(X)$, and*
2. *the set-valued function $\varphi(\omega, \cdot)$ is l.s.c. for each $\omega \in \Omega$.*

Then there exists a Carathéodory selection for φ.

Proof: See [16, Theorem 3.2]. ∎

Theorem 2.5 *The previous fact remains true if φ is a correspondence from $\Omega \times X$ into 2^Y, where Y is a separable Banach space and (1) and (2) are replaced by*

(1′) *$G_\varphi \in \alpha \otimes \beta(X) \otimes \beta(Y)$, and*

(2′) *the set-valued function $\varphi(\omega, \cdot)$ has an open graph for each $\omega \in \Omega$, i.e., for each $\omega \in \Omega$ the set $G_{\varphi(\omega,\cdot)} = \{(x, y) \in X \times Y : y \in \varphi(\omega, x)\}$ is open in $X \times Y$.*

Proof: See [17, Main Theorem]. ∎

Theorem 2.6 *Let Ω be a measurable space, $\{Y_i : i \in I\}$ (where I is a countable set) be a family of second countable topological spaces. Let $Y = \Pi_{i \in I} Y_i$. If for each $i \in I$, the correspondence $F_i: \Omega \to 2^{Y_i}$ is lower measurable, then the correspondence $F: \Omega \to 2^Y$, defined by $F(\omega) = \Pi_{i \in I} F_i(\omega)$, is also lower measurable.*

Proof: See [15, Proposition 2.3, p. 55]. ∎

Theorem 2.7 *Let Ω be a measurable space, X be a separable metric space and for each $i \in I$ (where I is a countable set) $F_i: \Omega \to 2^X$ is a lower measurable and closed-valued correspondence. If for each $\omega \in \Omega$ the set $F_i(\omega)$ is compact for at least one index $i \in I$, then the correspondence $F: \Omega \to 2^X$, defined by $F(\omega) = \bigcap_{i \in I} F_i(\omega)$, is lower measurable.*

Proof: See [15, Theorem 4.1, p. 58]. ∎

If (X, α), (Y, β) and (Z, Σ) are measurable spaces, $U \subset X \times Z$ and $f: U \to Y$, we call f *jointly measurable* if for every $B \in \beta$ we have $f^{-1}(B) = U \cap A$ for some $A \in \alpha \otimes \Sigma$. It is a standard result that if Z is a separable metric space, Y is a metric space and $f: X \times Z \to Y$ is such that for each fixed $x \in X$ the function