In our discussion, we shall need several results that will be listed in this section.

Theorem 2.1 *Let X be a topological space and Y be a linear topological space. If the correspondence $\varphi: X \to 2^Y$ is l.s.c., then the convex hull correspondence $\psi: X \to 2^Y$, defined by $\psi(x) = \operatorname{co}\varphi(x)$, is also l.s.c.*

Proof: See [19, Proposition 3.6, p. 366]. ∎

Theorem 2.2 *Let X be a topological space and let $\{Y_i : i \in I\}$ (where the set I can be finite or infinite) be a family of compact topological spaces. Let $Y = \Pi_{i \in I} Y_i$. If for each $i \in I$, the correspondence $F_i: X \to 2^{Y_i}$ is u.s.c. and closed valued, then the correspondence $F: X \to 2^Y$, defined by $F(x) = \Pi_{i \in I} F_i(x)$, is also u.s.c.*

Proof: See [11, Lemma 3, p. 124]. ∎

We now turn our attention to some measure theoretic facts. Let X and Y be topological spaces and let $\varphi: X \to 2^Y$ be a nonempty valued correspondence. A *continuous selection* for φ is a continuous function $f: X \to Y$ such that $f(x) \in \varphi(x)$ for all $x \in X$.

Let (Ω, α) be a measurable space, Y be a topological space and $\varphi: \Omega \to 2^Y$ be a nonempty-valued correspondence. A *measurable selection* for φ is a measurable function $f: \Omega \to Y$ such that $f(\omega) \in \varphi(\omega)$ for all $\omega \in \Omega$.

We now define the concept of a Carathéodory selection which combines the notion of continuous selection and measurable selection.

Let (X, α) be a measurable space and let Y and Z be topological spaces. Let $\varphi: X \times Z \to 2^Y$ be a (possibly empty-valued) arbitrary correspondence. Let $U = \{(x, z) \in X \times Z : \varphi(x, z) \neq \emptyset\}$. A *Carathéodory selection* for the correspondence φ is a function $f: U \to Y$ such that:

1. $f(x, z) \in \varphi(x, z)$ for all $(x, z) \in U$;
2. the function $f(x, \cdot)$ is continuous on $U^x = \{z \in Z : (x, z) \in U\}$ for each $x \in X$; and
3. the function $f(\cdot, z)$ is measurable on $U^z = \{x \in X : (x, z) \in U\}$ for each $z \in Z$.

If (X, α) and (Y, β) are measurable spaces and $\varphi: X \to 2^Y$ is a correspondence, then φ is said to have a *measurable graph* if G_φ belongs to the product σ-algebra $\alpha \otimes \beta$. We are usually interested in the situation where (X, α) is a measurable space, Y is a topological space and $\beta = \beta(Y)$ is the Borel σ-algebra of Y. For a correspondence φ from a measurable space into a topological space, if we say that φ has a measurable graph, it is understood that the topological space is endowed with its Borel σ-algebra (unless specified otherwise). In the same setting as above, i.e., (X, α) a measurable space and Y a topological space, φ is said to be *lower measurable* if $\{x \in X : \varphi(x) \cap V \neq \emptyset\} \in \alpha$ for every V open in Y. The following facts will be useful in the sequel.