the world and $X = \Pi_{i \in I} X_i$), an information set S_i (where S_i is a partition of Ω), and a prior q_i (i.e., a probability measure on Ω). In this setting the corresponding natural extension of Nash's equilibrium concept is that of a Bayesian equilibrium. In particular, if we denote by $E_i(\omega)$ the event in S_i containing the realized state of nature $\omega \in \Omega$, then each agent will choose a strategy which maximizes expected utility conditional on his/her own event $E_i(\omega)$.

Note that in this Bayesian game the conditional expected utility of each player is a random function, i.e., depends on the states of nature of the world and on the strategies. Hence, in essence the problem of the existence of a Bayesian equilibrium is converted to a random equilibrium problem, simply by thinking of the conditional expected utility of each player as his/her random payoff function of some random game. It is exactly for this reason that in certain cases the existence of a Bayesian equilibrium for a Bayesian game follows directly from the existence of a random equilibrium for a random game. The latter result seems to be quite interesting. Specifically, in view of recent work mentioned above, it is important to delineate conditions under which such equilibria exist.

As the deterministic results of Nash, Fan and Browder are based on deterministic fixed point theorems, the proofs of our random equilibrium existence results are based on random fixed point theorems. The idea behind the need of a random fixed point can be intuitively grasped simply by noting that with random payoff functions the best reply correspondence becomes random as well, and therefore a random extension of the Kakutani–Fan–Glicksberg Fixed Point Theorem seems to be required. To this end, we prove a random version of K. Fan's Coincidence Theorem [12, Theorem 6, p. 238], which gives as corollary a random version of the Kakutani–Fan–Glicksberg Fixed Point Theorem. In addition, we employ Aumann-type measurable selection theorems and some recent Carathéodory-type selection results proved in [16, 17].

The paper is organized as follows: Section 2 contains several preliminary results of measure theoretic character. Moreover, a random version of Fan's Coincidence Theorem is established. The main results of the paper are stated in Section 3, 4, and 5. Section 6 contains a discussion of the related literature on games with incomplete information. Finally, concluding remarks are given in Section 7.

2. Mathematical preliminaries

Let X and Y be sets. The graph G_φ of the set-valued function (or correspondence) $\varphi: X \to 2^Y$ is the set $G_\varphi = \{(x, y) \in X \times Y : y \in \varphi(x)\}$. If X and Y are topological spaces, a correspondence $\varphi: X \to 2^Y$ is said to have an *open graph* if the set G_φ is open in $X \times Y$. A correspondence $\varphi: X \to 2^Y$ is said to be *lower semicontinuous* (l.s.c.) if the set $\{x \in X : \varphi(x) \cap V \neq \emptyset\}$ is open in X for every open subset V of Y; and *upper semicontinuous* (u.s.c.) if the set $\{x \in X : \varphi(x) \subset V\}$ is open in X for every open subset V of Y. It can be easily checked that if a correspondence has an open graph, then it is l.s.c., but the converse is not true; see [32, p. 237].