interesting applications in game theory and mathematical economics (see for instance K. J. Arrow and G. Debreu [2] or G. Debreu [10]). Generalizations of Nash's equilibrium existence theorem to games where strategy sets were subsets of arbitrary Hausdorff linear topological spaces, were obtained by K. Fan [13] and F. E. Browder [8] among others. The results of Fan and Browder were proved by means of infinite dimensional fixed point theorems. Subsequently to the above work, research in economics (see for instance W. J. Shafer and H. F. Sonnenschein [31]) necessitated further generalizations of Nash's equilibrium existence result, to games where each player is equipped with a preference correspondence (instead of a payoff function), which need not be transitive or complete and therefore need not be representable by a utility function. The latter work was motivated by empirical results which indicated that in many instances agents' behavior is not necessarily transitive.

A common characteristic of all the above results is that they are deterministic, i.e., players cannot accommodate any kind of uncertainty or randomness in their responses to potential changes in their primitive environment. In reality, however, there are many factors which go beyond the control of players and cannot be influenced by their actions. In that sense, it seems natural to assume that players' payoff functions depend not only on the strategies, but on the states of nature of the world as well. In other words, payoff functions can be random. This is the type of the game we shall consider in this paper. Of course with the random payoff functions the equilibrium strategy vector will be random as well, and therefore the equilibrium will change from one state of the environment to another.

It is the purpose of this paper to prove random equilibrium existence results for quite general random games. In particular, as in W. J. Shafer and H. F. Sonnenschein [31] and N. C. Yannelis and N. D. Prabhakar [32], instead of assigning each player a random payoff or utility function, we equip each player with a random preference correspondence which need not be representable by a random utility function. It should be noted, however, that our random equilibrium results, provide as corollaries random versions of the theorems of Nash, Fan, and Browder. Moreover, we show that these random equilibrium theorems can be used to obtain equilibrium existence results for games with incomplete information, and in particular, for Bayesian games. The main reference for the latter type of games is J. C. Harsanyi's seminal paper [14]. Recently there is a growing literature on this subject. In particular, Bayesian games have found very interesting applications in economic theory, e.g., R. J. Aumann [3], R. Myerson [21], T. Palfrey and S. Srivastava [23, 24], J. Peck and K. Shell [25] and A. Postlewaite and D. Schmeidler [26] among others.[2]

As in [3, 14, 21, 23, 24, 25] by the term "Bayesian games" we mean games, where each player i is characterized by a strategy set X_i, a random utility function u_i defined on the product space $\Omega \times X$ (where Ω is the set of states of

[2]However, no equilibrium existence results are contained in these papers. E. J. Balder [6], A. Mas-Colell [18], P. Milgrom and R. Weber [20] and R. Radner and R. Rosenthal [27] have provided existence of equilibrium theorems for games with incomplete information, but their approach is different from ours. We shall discuss the work of these authors in Section 3.