Equilibrium Points of Non-Cooperative Random and Bayesian Games

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We provide random equilibrium existence theorems for non-cooperative random games with a countable number of players. Our results yield as corollaries generalized random versions of the ordinary equilibrium existence result of J. Nash [22]. Moreover, they can be used to obtain equilibrium existence results for games with incomplete information, and in particular Bayesian games. In view of recent work on applications of Bayesian games and Bayesian equilibria, the latter results seem to be quite useful since they delineate conditions under which such equilibria exist.

1. Introduction

A finite game consists of a set of players $I = \{1, 2, \ldots, n\}$ each of whom is characterized by a strategy set X_i and a payoff (utility) function

$$u_i\colon \prod_{j \in I} X_j \to R.$$

An equilibrium for this game is a strategy vector such that no player can increase his/her payoff by deviating from his/her equilibrium strategy, given that the other players use their equilibrium strategies, i.e., $x^* \in \Pi_{i \in I} X_i$ is an equilibrium if

$$u_i(x^*) = \max_{y_i \in X_i} u_i(x_1^*, \ldots, x_{i-1}^*, y_i, x_{i+1}^*, \ldots, x_n^*)$$

for all $i \in I$. The above game form and the notion of equilibrium were both introduced in a seminal paper by J. Nash [22].[1] In that same paper Nash proved by means of the Brouwer Fixed Point Theorem, the existence of an equilibrium for the above game, where strategy sets were subsets of R^ℓ, i.e., the ℓ-fold Cartesian product of the set of real numbers R. The work of Nash has found very

*We would like to thank Roko Aliprantis, Erik Balder, Kim Border, Taesung Kim, M. Ali Khan and David Schmeidler for comments, discussions and suggestions. Of course we are responsible for any remaining shortcomings.

[1]Notice that this notion of equilibrium is non-cooperative. No communication between players is allowed.