(1) $x \leqq y$ implies $x+z \leqq y+z$ (for all $x, y, z \in L$),
(2) $x \leqq y$ implies $tx \leqq ty$ (for all $x, y, \in L$, all real numbers $t \geqq 0$),
(3) every pair of elements $x, y \in L$ has a supremum (least upper bound) $x \vee y$ and an infimum (greatest lower bound) $x \wedge y$,
(4) $|x| \leqq |y|$ implies $\|x\| \leqq \|y\|$ (for all x, $y \in L$).

Here we have written, as usual, $|x| = x^+ + x^-$ where $x^+ = x \vee 0$, $x^- = (-x) \vee 0$; we call x^+, x^- the *positive* and *negative parts* of x (respectively) and $|x|$ the *absolute value* of x. We recall that $x = x^+ - x^-$, and that $x^+ \wedge x^- = 0$. We say that $x \in L$ is *positive* if $x \geqq 0$; we write L^+ for the set of all positive elements of L and refer to L^+ as the *positive cone* of L.

The Banach lattice structure on L induces on the dual space L' the structure of a Banach lattice, where $f \leqq g$ in L' means $f(x) < g(x)$ for each $x \in L'$.

If x is a positive element of L, then by the *order interval* $[0, x]$ we mean the set

$$[0, x] = [y: y \in L, 0 \leqq y \leqq x\}.$$

In any Banach lattice L, order intervals are norm closed (and thus weakly closed), convex and bounded. If L is a dual lattice, order intervals are also weak-star closed (and thus weak-star compact).

We shall say that an element x of L is *strictly positive* (and write $x \gg 0$) if $\phi(x) > 0$ whenever ϕ is a positive non-zero element of L^+. (Strictly positive elements are sometimes called *quasi-interior* to L^+.) An equivalent characterization is that the element x in L is strictly positive if and only if the sequence $\{nx \wedge y\}$ converges in norm to y (as n tends to infinity) for each y in L^+. We note that if the positive cone L^+ of L has a non-empty (norm) interior, then the set of strictly positive elements coincides with the interior of L^+. However, many Banach lattices contain strictly positive elements even though the positive cone L^+ has an empty interior.

Basic examples of Banach lattices include:

(i) the Euclidean space $\mathbb{R}^N$,
(ii) the space l_p (for $1 \leqq p < \infty$) of real sequences (a_n) for which the norm $\|(a_n)\|_p = (\sum |a_n|^p)^{1/p}$ is finite,
(iii) the space $L_p(\Omega, R, \mu)$ of measurable functions f on the measure space $(\Omega. R, m)$ for which the norm $\|f\|_p = (\int_\Omega |f|^p \, d\mu)^{1/p}$ is finite (as usual, we identify two functions if they agree almost everywhere),
(iv) the space l_∞ of bounded real sequences (with the supremum norm),
(v) the space $L_\infty(\Omega, R, m)$ of essentially bounded, measurable functions on a measure space [with the essential supremum norm, and the same identification as in (iii)].