A *normed vector space* is a real vector space E equipped with a norm $\|\cdot\|$: $E \to [0, \infty)$ satisfying:

(i) $\|x\| \geqq 0$ for all x in E, and $\|x\|=0$ if and only if $x=0$,
(ii) $\|\alpha x\|=|\alpha|\,\|x\|$ for all x in E and all α in $\mathbb{R}$,
(iii) $\|x+y\| \leqq \|x\|+\|y\|$ for all x, y in E.

The *Banach space* is a normed vector space for which the metric induced by the norm is complete.

If E is a Banach space, then its *dual space* E' is a set of continuous linear functionals on E. The dual space E' is itself a Banach space, when equipped with the norm

$$\|\phi\| = \sup\{|\phi(x)| : x \in E, \|x\| \leqq 1\}.$$

In addition to the norm topologies on E and E', we shall make use of three other topologies. The *weak topology* $\sigma(E, E')$ on E is the topology of pointwise convergence when we regard elements of E as functionals on E'. That is, a net $\{x_\alpha\}$ in E converges weakly to an element $x \in E$ exactly when $\{f(x_\alpha)\}$ converges to $f(x)$ or each $f \in E'$. Similarly, the *weak-star topology* $\sigma(E', E)$ on E' is the topology of pointwise convergence when we regard elements of E' as functionals on E. Thus $f_\alpha \to f$ in the weak-star topology means that $f_\alpha(x) \to f(x)$ for each $x \in E$. Finally, the *Mackey topology* $\tau(E', E)$ on E' is the topology of uniform convergence on weakly compact, convex, symmetric, subsets of E.

It is a consequence of the Separation Theorem that the weak and norm topologies on E have the same closed convex sets and the same continuous linear functionals. The Mackey–Arens Theorem asserts that the weak-star and Mackey topologies on E' have the same closed convex sets and the same continuous linear functionals, and that the Mackey topology is the strongest locally convex vector space topology on E' with this property. By viewing elements of E as linear functionals on E' we obtain a canonical injection of E into E'' and we may identify E as the subspace of weak-star continuous linear functionals on E'.

Alaoglu's Theorem asserts that the closed unit ball of E' (and hence every weak-star closed, norm bounded set) is weak-star compact. Hence every net $\{\pi_\alpha\}$ in the ball of E' has a convergent/subnet. As a final comment, let us note for further use the following elementary lemma:

Lemma A. If $x_\alpha \to x$ in the norm topology of E, $\pi_\alpha \to \pi$ in the weak-star topology of E' and $\{\pi_\alpha\}$ is norm bounded, then $\pi_\alpha(x_\alpha) \to \pi(x)$.

Recall that a *Banach lattice* is a Banach space L endowed with a partial order $\leqq$ (i.e., $\leqq$ is a reflexive, antisymmetric, transmitive relation) satisfying: