(a) X_i is a closed, convex subset of L^+ containing 0,
(b) X_i is solid (i.e., if $x \in X_i$ then the order interval $[0, x]$ is contained in X_i),
(c) if $x \in X_i$ then $x + tv_i \in X_i$ for some $t > 0$.

Remark 2. Much of our analysis should go through in the context of a countable number of agents, provided the commodity space is l_1 or L_1. For more general commodity spaces, there seem to be additional serious difficulties. (See the Remark in section 5.)

Remark 3. Notice that, in the proofs of the Main Existence Theorem, the Price Lemma and Theorem 6.1, completeness of the norm of L was never used. The Main Existence Theorem therefore remains valid for incomplete normed vector lattices.

Remark 4. As we noted in section 4, the requirement that the aggregate initial endowment be strictly positive rules out the commodity space $M(\Omega)$, which has no strictly positive elements. However, our results can be extended to this case, if we strengthen the extreme desirability assumption slightly. Here is a sketch:

We will assume that for each consumer i, there is a commodity v_i which belongs to the order interval $[0, e]$ and is extremely desirable for i on some open set containing $[0, e]^N$. For each finite set $A = \{a_1, \ldots, a_k\}$ contained in $M(\Omega)^+$, we write $f_A = e + \sum a_i$, and consider the set M_A of measures which are absolutely continuous with respect to f_A; this is a closed sublattice of $M(\Omega)$ and f_A is strictly positive when viewed in the sublattice M_A. If we consider the restriction of $\mathcal{E}$ to M_A, and alter the initial endowment of each consumer to be $\tilde{e}_i = e_i + \varepsilon f_A$, for ε a small positive number, then we obtain an economy $\mathcal{E}_{A,\varepsilon}$ to which the Main Existence Theorem may be applied. The economies $\mathcal{E}_{A,\varepsilon}$ thus have quasi-equilibria; moreover, if π is a quasi-equilibrium price then $\sum(\pi(v_i)/\mu_i) \geqq 1$. If we now take limits (as A increases and ε tends to 0), we obtain a quasi-equilibrium $(\bar{x}_1, \ldots, \bar{x}_N; \bar{\pi})$ for $\mathcal{E}$ with $\sum(\bar{\pi}(v_i)/\mu_i) \geqq 1$. Hence, if $\mathcal{E}$ is irreducible, Lemma 7.2 can again be used to prove that $(\bar{x}_1, \ldots, \bar{x}_N; \bar{\pi})$ is an equilibrium.

Note that this argument produces an equilibrium price π in $M(\Omega)'$, not in $C(\Omega)$. If we want the price to lie in $C(\Omega)$, we must assume much more; see Yannelis–Zame (1984) for details.

Appendix

In this appendix we collect some basic information about Banach spaces in general and Banach lattices in particular. For further details, we refer the reader to Schaefer (1971, 1974).