economy with these endowments has an equilibrium (note that $\sum f_i^t = \sum e_i^\gamma$). Letting t tend to zero and taking limits yields our quasi-equilibrium $(x_1^\gamma, \ldots, x_N^\gamma; p^\gamma)$. The price p^γ belongs to F'_γ, and there is no loss in assuming that $\|p^\gamma\| = 1$. The Hahn–Banach Theorem now provides an element π^γ of L' which has norm one, and agrees with p^γ on F_γ.

We have thus constructed a net $\{(x_1^\gamma, \ldots, x_N^\gamma; \pi^\gamma)\}$ in $L \times L \cdots \times L \times L'$. Since $0 \leqq \sum x_i^\gamma = \sum e_i^\gamma \leqq e$ for each γ, the vectors x_i^γ all belong to the order interval $\{0, e]$, which is τ-compact. Moreover, since $\|\pi^\gamma\| = 1$, the functionals π^γ all belong to the unit ball of L', which is weak-star compact. Hence, passing to a subnet if necessary, we obtain vectors $\bar{x}_1, \ldots, \bar{x}_N$ in $(0, e]$ and a functional $\bar{\pi}$ in L' such that $\{x_i^\gamma\}$ converges to $\bar{x}_i$ (in the topology τ) and $\{\pi^\gamma\}$ converges to $\bar{\pi}$ (in the weak-star topology). Note that $\|\bar{\pi}\| \leqq 1$.

We are now going to show that $(\bar{x}_1, \ldots, \bar{x}_N; \bar{\pi})$ is a quasi-equilibrium for $\mathcal{E}$. Our first task is to show that $\bar{\pi}$ is not the zero functional; as we have emphasized, this is the crucial point. To do this, we first recall that $\|v_i^\gamma - v_i\|$ tends to zero, for each i, so that $\{\pi^\gamma(v_i^\gamma)\}$ converges to $\bar{\pi}(v_i)$, for each i (by Lemma A of the appendix). The Price Lemma gives us an estimate involving the $\pi^\gamma(v_i^\gamma)$, namely

$$\sum_{i=1}^{N} \frac{\pi^\gamma(v_i^\gamma)}{\mu_i(v_i^\gamma, (x_i^\gamma, \ldots, x_N^\gamma))} \geqq 1 \tag{*}$$

provided that we compute marginal rates of desirability in the economy $\mathcal{E}^\gamma$. However, let us note that $\pi^\gamma(v_i^\gamma) \geqq 0$ (since v_i^γ is extremely desirable and π^γ is a quasi-equilibrium price) and that marginal rates of desirability certainly do not increase if we compute them in $\mathcal{E}$ rather than in $\mathcal{E}^\gamma$. Hence the inequality (*) is valid if we compute marginal rates of desirability in the economy $\mathcal{E}$. Let us write

$$\mu_i = \inf\{\mu_i(v_i, (x_1, \ldots, x_N)) : (x_1, \ldots, x_N) \in \mathscr{A}\}.$$

By extreme desirability, $\mu_i > 0$. By Lemma 4.3,

$$\mu_i(v_i^\gamma, (x_1^\gamma, \ldots, x_N^\gamma)) \geqq \mu_i - \|v_i - v_i^\gamma\|.$$

As we have already noted, $\|v_i - v_i^\gamma\|$ tends to 0. If we combine this fact with the inequality (*) and our previous observation that $\{\pi^\gamma(v_i^\gamma)\}$ converges to $\bar{\pi}(v_i)$, we obtain

$$\sum_{i=1}^{N} \frac{\bar{\pi}(v_i)}{\mu_i} \geqq 1.$$

In particular, $\bar{\pi}$ is not the zero functional.