$$X_i^\alpha = X_i \cap F_\alpha = F_\alpha^+,$$

$$P_i^\alpha(x_1,\ldots,x_N) = P_i(x_1,\ldots,x_N) \cap F_\alpha.$$

We set $\mathcal{E}^\alpha = \{(X_i^\alpha, P_i^\alpha, e_i^\alpha)\}$; this is an economy in the finite-dimensional vector sublattice F_α of L. The next step is to find extemely desirable commodities.

For each i, fix a vector $v_i \in L^+$ which is extremely desirable (on the set $\mathscr{A}$) for consumer i. For each α, the distance from v_i to F_α^+ is acutally taken on (since F_α is finite-dimensional); i.e., we can choose vectors v_i^α in F_α so that

$$\|v_i^\alpha - v_i\| = \inf\{\|z - v_i\| : z \in F_\alpha^+\}.$$

For any $\varepsilon > 0$, we can use Lemma 7.2 to find a special configuration α such that $\|v_i^\alpha - v_i\| < \varepsilon$ for each i. On the other hand, the properties of our ordering require that $\operatorname{dist}(v_i^\alpha, F_\beta^+) \leqq 4 \cdot 2^{-n_\alpha}\|v_i^\alpha\| \leqq 4 \cdot 2^{-n_\alpha}(\|v_i\| + \varepsilon)$. whenever $\beta \geqq \alpha$. Hence there is a vector in F_β^+ whose distance to v_i is at most $4 \cdot 2^{-n_\alpha}(\|v_i\| + \varepsilon) \times \varepsilon$. The definition of v_i^β now yields that

$$\|v_i^\beta - v_i\| \leqq 4 \cdot 2^{-n_\alpha}(\|v_i\| + \varepsilon) + \varepsilon$$

whenever $\beta \geqq \alpha$. We conclude that, for each i, $\|v_i^\gamma - v_i\|$ tends to 0 along the directed set D of special configurations. It follows immediately from Lemma 3.2 that (for each i), v_i^γ is extremely desirable for consumer i (on the set A), provided γ is sufficiently large. Since v_i^γ bellongs to F_γ^+, it is certainly extremely desirable for consumer i on $A \cap F_\gamma^+$, which includes the set of feasible allocations for the economy $\mathcal{E}^\gamma$ (provided that γ is sufficiently large).

We now want to apply the equilibrium existence result of Shafer (1976, Theorem 2 and Remarks) to conclude that each of the economies $\mathcal{E}^\gamma$ has a quasi-equilibrium $(x_1^\gamma,\ldots,x_N^\gamma; p^\gamma)$, for γ sufficiently large. To do so, we first note that the existence of extremely desirable commodities implies that preferences are locally non-satiated on the set of feasible allocations. Shafer's continuity assumptions follow from continuity of the preferences P_i, together with the fact that all Hausdorff vector space topologies on a finite-dimensional vector space coincide. The remaining conditions of Shafer's Theorem are easily verified, except for the requirement that the initial endowments lie in the interiors of the consumption sets. To remedy this small difficulty, we choose a real number t with $0 < t < 1$ and define new endowments

$$f_i^t = (1-t)e_i^\gamma + (t/N)\sum_{j=1}^N e_j^\gamma.$$

These new endowments do lie in the interiors of the consumption sets, so the