initial endowments of our approximating subeconomies. The integers n_α will play a role when we make the family of all special configurations into a directed set.

Our first task is to show that many special configurations exist. Given a finite subset A of L^+ we cannot generally find a special configuration $\alpha=(F_\alpha, n_\alpha, e_1^\alpha, \dots, e_n^\alpha)$ with $A \subset F_\alpha$, but we can come as close as we wish.

Lemma 7.2 Let A be a finite subset of L^+ and $\varepsilon>0$ a positive number. Then there is a special configuration $\alpha=(F_\alpha, n_\alpha, e_1^\alpha, \dots, e_N^\alpha)$ such that $1/n_\alpha<\varepsilon$ and

$$\operatorname{dist}(a, F_\alpha^+)=\inf\{\|a-z\|: z\in F_\alpha^+\}<\varepsilon$$

for each element a of A.

Proof. Write $A=\{a_1,\dots,a_M\}$, and choose an integer s with $1/s<\varepsilon$. Since e is strictly positive, $\lim_{n\to\infty}(ne\wedge a_j)=a_j$ (for each j). Hence we can find an integer R so large that

$$\|(Re\wedge a_j)-a_j\|<\varepsilon/2$$

for each j. Set $b_j=R^{-1}(Re\wedge a_j)$ for each j, and note that $0\leqq b_j\leqq e$ for each j. Hence, we may set $\delta=\min(1/s, \varepsilon/2M)$ and apply Theorem 6.1 to obtain a finite-dimensional vector sublattice K of L and elements $e_1^*,\dots,e_N^*$, $b_1^*,\dots,b_M^*$ of K such that

(1) $0\leqq e_i^*\leqq e_i$ and $\|e_i^*-e_i\|<\delta$ for each i,
(2) $0\leqq b_j^*\leqq b_j$ and $\|b_j^*-b_j\|<\delta$ for each j,
(3) $\sum_{i=1}^N e_i^*$ is strictly positive in K.

Then $\alpha=(K, s, e_1^*,\dots,e_N^*)$ is a special configuration. Moreover, for each j, Rb_j^* belongs to K and our choice of b_j and the triangle inequality imply that $\|a_j-Rb_j^*\|<\varepsilon$. Hence the special configuration α has the required properties, and the proof is complete. □

We will write D for the set of all special configurations. We wish to use D to index nets of quasi-equilibria; to do so, we must define an ordering on D.

Given two special configurations $\alpha=(F_\alpha, n_\alpha, e_1^\alpha,\dots,e_N^\alpha)$ and $\beta=(F_\beta, n_\beta, e_1^\beta,\dots,e_N^\beta)$, we will write $\alpha<\beta$ provided that $n_\alpha<n_\beta$ and

$$\operatorname{dist}(z, F_\beta^+)\leqq 2^{-n_\alpha}\|z\|$$

for each $z\in F_\alpha^+$. This relation is not transitive, but it is acyclic; i.e., there is no finite sequence $\alpha_1,\alpha_2,\dots,\alpha_k$ of special configurations such that $\alpha_1<\alpha_2<\cdots<\alpha_k<\alpha_1$ (because we cannot have $n_{\alpha_1}<n_{\alpha_2}\cdots<n_{\alpha_k}<n_{\alpha_1}$). Hence