a subnet of $\{z_G]$, and hence also converges (in the topology τ) to $\bar{z}$. Since τ is a vector space topology, this implies that $\{z_G - z_{F_0}\}$ converges (in the topology τ) to $\bar{z} - z_{F_0}$. Since $z_G - z_{F_0}$ lies in the τ-compact order interval $[0, z - z_{F_0}]$, so does $\bar{z} - z_{F_0}$. In particular, $\bar{z} \geqq z_{F_0}$ for each F_0 in $\mathscr{F}$.

To see that $\bar{z} = \sup\{z_F\}$ we consider any w in L such that $w \geqq z_F$ for each F; we must show that $w \geqq \bar{z}$. Since $w \geqq z_F$ for each F, it follows in particular that $w \geqq z_G$ for each G in $\mathscr{J}$ and hence (as above) that $w \geqq \bar{z}$, as desired. Hence $\bar{z} = \sup\{z_F\}$, as asserted.

Finally, since $z_F = \sup\{z_\lambda : \lambda \in F\}$, it is clear that $\sup\{z_F\} = \sup\{z_\lambda : \lambda \in \Lambda\}$, so that $\{z_\lambda\}$ has a supremum. This completes the proof of Lemma 6.2. □

We now turn to the proof of Theorem 6.1.

Proof of Theorem 6.1. We may assume without loss that $0 < \delta < 1$. We set $e = \sum_{i=1}^N e_i$, and consider the *principal order ideal*

$$L_e = \{y \in L : -re \leqq y \leqq re \quad \text{for some integer } r\}.$$

According to Schaefer (1974, pp. 102, 104), L_e is an abstract M-space with e as order unit. and hence is order-isomorphic to the space $C(\Omega)$ of continuous real-valued functions on some compact Hausdorff space Ω. Moreover, under this isomorphism, the element e in L_e corresponds to the function on Ω which is identically equal to 1. Since L is order-complete, so is L_e. Hence by Schaefer (1974, p. 108) the space Ω is *Stonian*; i.e., the closure of every open subset of Ω is open.

In what follows, it will be convenient to suppress the isomorphism between L_e and $C(\Omega)$, and simply identify them. We will thus write $e_i(w)$ for the value of e_i at w, etc. We continue, however, to write $\|e_i\|$ for the norm of e_i in L, etc.

Choose any δ' with $0 < \delta' < \min(1/N, \delta/\|e\|)$. Since Ω is compact and each function e_i, b_j is continuous (hence uniformly continuous) we can find a covering of Ω by open sets $U_1, \dots, U_k$ such that, for each i, j, $|e_i(w) - e_i(w')| < \delta'$ and $|b_j(w) - b_j(w')| < \delta'$ whenever w, w' belong to the same set U_l. Let $\bar{U}_l$ denote the closure of u_l, and set $V_1 = \bar{U}_1$, $V_2 = \bar{U}_2 - V_1$, $V_3 = \bar{U}_3 - (V_1 \cup V_2)$, etc. Since Ω is Stonian, the sets $V_1, \dots, V_k$ form a cover of Ω by open and closed sets. (We may, without loss, assume that $V_l \neq 0$ for each l.) Moreover, for each i, j, $|e_i(w) - e_i(w')| \leqq \delta'$ and $|b_j(w) - b_j(w')| \leqq \delta'$ whenever w, w' belong to the same set V_l.

Now define K to be the subspace of $C(\Omega) = L_e$ consisting of functions which are constant on each of the sets V_l. It is evident that K is a finite-dimensional vector sublattice of $C(\Omega) = L_e$ (and hence of L). In fact, a basis for K consists of the characteristic functions χ_{V_l}, $l = 1, 2, \dots, k$.

For each i, j, l, let c_{il} be the minimum of the continuous function e_i on the compact set V_l, and let d_{jl} be the minimum of b_j on V_l. Set