inclusion' instead. The final difficulty lies in showing that the limiting allocation is an equilibrium allocation, since it need not lie in any of the approximating economies; we take care of this by another approximation argument.

5. The Price Lemma

As we discussed in the previous section, the crucial issue in our argument is that the limiting price we construct must be different from zero. To achieve this, we shall make use of the following lemma, which formalizes a very natural economic intuition: at equilibrium, commodities which are very desirable cannot be cheap.

Price Lemma. *Let L be a Banach lattice, let $\mathcal{E}=\{(X_i, P_i, e_i): i=1,2,\dots,N]$ be an economy in L and let $(x_1,\dots,x_N;\pi)$ be a quasi-equilibrium*[6] *for $\mathcal{E}$ with $\|\pi\|=1$. Assume that*

(1) $X_i=L^+$ for each i,
(2) $e=\sum_{i=1}^N e_i$ is strictly positive,
(3) for each i, there is a commodity $v_i \in L^+$ such that the marginal rate of desirability $\mu_i(v_i,(x_1,\dots,x_N))$ is not zero.

Then

$$\sum_{i=1}^{N} \frac{\pi(v_i)}{\mu_i(v_i,(x_1,\dots,x_N))} \geqq 1.$$

Proof. If this is not so, we will show how to construct vectors y_i which are all preferred to the given allocation and have the property that, for at least one agent i, y_i is cheaper than x_i; this will violate the quasi-equilibrium conditions.

To this end, we write $\mu_i=\mu_i(v_i,(x_1,\dots,x_N))$, and suppose that $\sum(\pi(v_i)/\mu_i)<1$. Since $\|\pi\|=1$, there is a vector $w \in L$ such that $\|w\|<1$ and $\pi(w)>\sum(\pi(v_i)/\mu_i)$. Write $w=w^+-w^-$. Since $e=\sum e_i$ is strictly positive, the sequences $\{ne \wedge w^+\}_{n=1}^{\infty}$ $\{ne \wedge w^-\}_{n=1}^{\infty}$ and converge in norm to w^+ and w^- respectively. Since π is norm continuous, we can choose a positive integer k so large that $\pi((ke \wedge w^+)-(ke \wedge w^-))>\sum(\pi(v_i)/\mu_i)$. Write

$$z=(ke \wedge w^+)-(ke \wedge w^-).$$

[6]Notice that the lattice L need not admit a compatible topology and that the preferences need not enjoy any continuity properties. In this generality, quasi-equilibria need not exist (in which case the Price Lemma is certainly true). Of course, in our applications we will make additional assumptions about L and P_i, but the Price Lemma seems to be of interest in itself, so we choose to give a proof in this more general setting.