F of $L=L_\infty$ which contain the initial endowments. Standard results imply that each of the economies $\mathcal{E}^F$ has an equilibrium $(x_1^F,\dots,x_N^F;p^F)$ with $p^F\in F'$. Since Bewley assumes that preferences are monotone, it is necessarily the case that $p^F\geqq 0$ and there is no loss of generality in assuming that $\|p^F\|=1$. Since e is strictly positive, it is in fact an interior point of the positive cone of $L=L_\infty$. The Krein–Rutman Theorem then allows us to extend p^F to an element π^F of $L'=L'_\infty$ with $\pi^F\geqq 0$ and $\|\pi^F\|=1$. The net of equilibria $(x_1^F,\dots,x_N^F;\pi^F)$ has a subnet which converges (in the respective weak-star topologies) to $(\bar{x}_1,\dots,\bar{x}_N;\bar{\pi})$. Since the functionals π^F are positive and have norm 1, non-emptiness of the interior of the positive cone implies that the (weak-star) limit functional $\bar{\pi}$ is also positive and also has norm 1. In particular, $\bar{\pi}$ is not the zero functional. It now follows that $(\bar{x}_1,\dots,\bar{x}_N;\bar{\pi})$ is an equilibrium for $\mathcal{E}$.

This argument depends crucially *both* on the assumption that preferences are monotone *and* on the assumption that the positive cone of the Banach lattice L has a non-empty interior; it will not work if *either* of these assumptions is dropped. The problem is that we must be sure that the limit price $\bar{\pi}$ is not identically zero. If preferences are not monotone, we cannot be sure that the prices p^F (and hence their extensions π^F) are positive. Since the functionals π^F only converge to $\bar{\pi}$ in the weak-star topology, however, there is then no reason to suppose that $\bar{\pi}$ is not identically zero. (This can happen in the dual of *any* infinite-dimensional Banach space, including L_∞.) On the other hand if the positive cone of L has an empty interior, then the limit functional $\bar{\pi}$ may again be zero – even if all the functionals π^F are positive and have norm 1.

The purpose of this discussion is to point out that the crucial issue is to guarantee that the limit functional $\bar{\pi}$ is not identically zero. The central idea of our proof is to consider, not finite-dimensional subspaces of L, but rather finite-dimensional vector sublattices. For vector sublattices, we can use the extremely desirable commodities v_i to obtain an estimate (which we call the Price Lemma, and isolate in section 5) which will, in the limit, guarantee that $\bar{\pi}$ is not identically zero. However, this approach creates a multitude of its own difficulties. The first difficulty is that, in general, a Banach lattice need not have 'enough' finite-dimensional vector sublattices; we take care of this in section 6 by showing that the existence of a compatible topology implies the existence of 'many' finite-dimensional vector sublattices. The second difficulty is that, even with an abundance of finite-dimensional vector sublattices, we cannot be sure of finding *any* finite dimensional vector sublattices which contain the initial endowments; we take care of this by constructing economies in the finite-dimensional vector sublattices which are approximations of the original economy, rather than restrictions of it. The third difficulty is that the family of finite-dimensional vector sublattices is not directed by inclusion; we take care of this by directing them by 'approximate