(2) *the aggregate initial endowment* $e=\sum_{i=1}^{N} e_i$ *is strictly positive,*
(3) $x_i \notin \operatorname{con} P_i(x_1,\ldots,x_N)$, *for each agent i and each point* $(x_1,\ldots,x_N)$ *in* $(L^+)^N$.
(4) *for each agent i, there is a commodity* $v_i \in L^+$ *which is extremely desirable for agent i on the set*

$$\mathscr{A}=\left\{x=(x_1,\ldots,x_N): x\in(L^+)^N,\ \sum_{i=1}^{N} x_i \leqq e\right\}$$

of feasible allocations,
(5) *each of the preference relations* P_i *is* (τ, norm)*-continuous.*

Then $\mathcal{E}$ *has a quasi-equilibrium* $(\bar{x}_1,\ldots,\bar{x}_N;\bar{\pi})$ *with the price* $\bar{\pi}$ *belonging to* L'. *If* $\mathcal{E}$ *is irreducible, then every quasi-equilibrium is an equilibrium.*[4,5]

In concrete settings, the choice of compatible topology will be dicated by the underlying commodity space L. For instance, if $L=l_p(1\leqq p<\infty)$ [the space of real sequences (a_n) such that $\|(a_n)\|_p=(\sum|a_n|^p)^{1/p}<\infty$], then the norm topology itself is compatible [since order intervals in l_p are norm compact – see Yannelis and Zame (1984) for a proof]. If $L=L_p(1\leqq p<\infty)$ (the space of equivalence classes of pth power integrable functions on a measure space) then the weak topology is compatible [since order intervals are weakly compact – see Schaefer (1974, pp. 90–92, 119)]. If $L=l_\infty$ or L_∞, then the weak-star topology is compatible (since order intervals are weak-star closed and bounded, hence weak-star compact by Alaoglu's Theorem). With the appropriate choice of compatible topology, the Main Existence Theorem simply applies verbatim for economies in any of these commodity spaces.

If the commodity space is $M(\Omega)$ (the space of regular Borel measures on the compact space Ω), the weak-star topology is again compatible. However, the Main Existence Theorem may not be applicable since it requires that the aggregate initial endowments be strictly positive, and $M(\Omega)$ need not have any strictly positive elements. However, it is possible to adapt our result to cover this case – see Remark 4 of section 8 for details.

The formal proof of the Main Existence Theorem is long and involved; we defer it to the following sections. At this point, however, it is appropriate to give an overview of the proof.

It is helpful to recall the argument used by Bewley (1972) (and generalized by others) for the case $L=L_\infty$. In sketch, the strategy of Bewley's proof is to consider the restriction $\mathcal{E}^F$ of the economy $\mathcal{E}$ to finite-dimensional subspaces

[4]We have required extreme desirability on the set of feasible allocations, rather than all of $(L^+)^N$, since that is all we shall need, and it is a bit easier to verify in practice. See Yannelis and Zame (1984) for example.

[5]Note that the price π is norm continuous but need not be continuous in the topology τ. Indeed, τ need not even admit any non-zero continuous linear functionals.