rate of desirability $\mu_i(v,x)$ depends both on the bundle v and on the allocation x. As a function of x, $\mu_i(v,x)$ need not be continuous, but as a function of v we have the following easy estimate which we shall need later:

Lemma 3.2. Let v and w belong to L^+ and let x belong to $\prod_{j=1}^N X_j$. Then:

$$\mu_i(w,x) \geqq \mu_i(v,x) - \|v-w\|.$$

Proof. Let t be a real number with $0<t\leqq 1$ and let σ be an element of L with $\sigma \leqq x_i+tw$ and $\|\sigma\| \leqq t\mu_i(v,x) - t\|v-w\|$; we wish to show that $x_i+tw-\sigma \in P_i(x)$. Write $x_i+tw-\sigma = x_i+tv-(\sigma+tv-tw)$ and observe that $\sigma+tv-tw \leqq x_i+tv$ (since $\sigma \leqq x_i+tw$ and $tv \geqq 0$) and that

$$\begin{aligned}\|\sigma+tv-tw\| &\leqq \|\sigma\| + t\|v-w\| \\ &\leqq t\mu_i(v,x) - t\|v-w\| + t\|v-w\|,\end{aligned}$$

so that $\|\sigma+tv-tw\| < t\mu_i(v,w)$. Of course this means that $x_i+tw-\sigma = x_i+tv-(\sigma+tv-tw)$ belongs to $P_i(x)$ as desired. □

4. The Main Existence Theorem

In this section we formulate a very general existence result from which we can easily derive concrete applications. We begin with a definition.

Definition. A Hausdorff topology τ on the Banach lattice L will be called *compatible* if

(a) τ is weaker than the norm topology of L,
(b) τ is a vector space topology (i.e., the vector space operations on L are continuous in the topology τ),
(c) all order intervals $[0,z]$ in L are τ-compact.

Note that we do *not* assume that the lattice operations in L are continuous in the topology τ. In concrete applications, the topology τ will vary according to the underlying Banach lattice L; it may be the norm topology itself, or the weak topology, or the weak-star or Mackey topology (if L is a dual space).

Our basic existence result is the following:

Main Existence Theorem. Let $\mathcal{E} = \{(X_i, P_i, e_i)$: $i=1,2,\ldots,N\}$ be an economy in the Banach lattice L, and let τ be a compatible topology on L. Assume that:

(1) $X_i = L^+$ for each agent i,