The vector v is extremely desirable (for agent i) with marginal rate of desirability at least μ, if for each x in $(L^+)^N$; it is the case that $y \in P_i(x)$ whenever y belongs to $(C+x_i) \cap L^+$. In other words, extreme desirability means that the portion of the forward cone $C+x_i$ which belongs to the consumption set of consumer i is contained in the set of vectors preferred to x_i (keeping other components of x fixed).

By way of comparison, Mas-Colell (1983) says that the (transitive, complete, convex) preference relation $\succsim_i$ is *uniformly proper* if there is a vector v in L^+ and a positive real number μ such that $(x_i - tv + \sigma) \not\succsim_i x$ whenever $x_i \in L^+, t>0$, $\sigma \in L$ and $\|\sigma\| < t\mu$. Since it is automatically the case that $(x_i - tv + \sigma) \not\succsim_i x$ if $(x_i - tv + \sigma) \notin L^+$, this is equivalent to saying that $[(-C+x_i) \cap L^+] \cap \{y_i: y_i \succsim_i x_i\} = \varnothing$. Thus uniform properness means that the portion of the backward cone $-C+x_i$ which belongs to the consumption set of consumer i is disjoint from the set of vectors preferred to x_i.

It should be evident, then, that the existence of extremely desirable commodities is simply the non-transitive analog of uniform properness. In fact, it may be shown [see Yannelis and Zame (1984) for the easy argument] that – for transitive, complete, convex, non-interdependent preferences – the two conditions are equivalent. All of Mas-Colell's comments on the meaning of uniform properness thus apply to extreme desirability; we shall not repeat them here. Nor shall we repeat Mas-Colell's example which shows that, without uniform properness (i.e., in the absense of extremely desirable commodities), an economy may fail to have an equilibrium. It does, however, seem natural to give one example to illustrate the economic meaning of extremely desirable commodities.

Example 3.1. Let $(\Omega, \mathscr{R}, m)$ be a measure space with m a positive measure such that $m(\Omega)=1$. We wish to think of Ω as representing the set of possible states of the world, so that $m(E)$ is the probability that the true state of the world is one of the states in the set E, with $E \in \mathscr{R}$. We interpret a function $f \in L_1^+$ as representing the allocation of a single resource over all possible states of the world so that $\|f\| = \int_\Omega f \, dm$ is the consumer's expected allocation of this one resource. Take v to be the function which is identically equal to 1, so that v represents a guarantee of one unit of the resource no matter what the true state of the world is. If $\|\sigma\| = \int_\Omega |\sigma| \, dm$ is small in comparison with t, then $x + tv - \sigma$ represents a guaranteed gain of t units of the resource in every state of the world, and a loss of an amount which, although perhaps large in some states of the world, is expected to be small (in comparison with t). To say that the bundle v is extremely desirable (with some marginal rate of desirability) is thus to place a bound on the degree to which the consumer is 'risk-preferring'.

One further comment of a mathematical nature. Notice that the marginal