(The restriction $\sigma \leqq x_i + tv$ guarantees that $x_i + tv - \sigma \geqq 0$ so $x_i + tv - \sigma \in X_i = L^+$.) It is easily checked that $\Gamma_v^i(x)$ is a closed interval containing 0 and bounded above by $\|v\|$.

Definition The *marginal rate of desirability of v (for agent i) at x* is

$$\mu_i(v, x) = \max\{\mu : \mu \in \Gamma_v^i(x)\}.$$

If A is a subset of $\prod X_j$, we say the vector v is *extremely desirable (for agent i) on the set A if*

$$\inf\{\mu_i(v, x) : x \in A\} > 0.$$

Finally, v is *extremely desirable (for agent i)* if it is extremely desirable on $(L^+)^N$.

Informally, v is extremely desirable if agent i would prefer to trade any bundle σ for an additional increment of the bundle v, provided that the size of σ (measured by $\|\sigma\|$) is sufficiently small compared to the increment of v (measured by t). We stress that – even in those contexts where it makes sense to speak of 'pure commodities' – the vector v need not be a pure commodity; but rather a commodity bundle. Evidently, extreme desirability is a kind of bound on the relative marginal rates of substitution, where we compare v to all other bundles.

As will become clear in the following sections, existence of extremely desirable commodities (for each agent) is precisely the additional assumption we need to obtain existence of equilibria, so its seems valuable to discuss the meaning of this assumption in some detail.

Let us observe first of all that if the preference relation P_i is strictly monotone [in the sense that $x_i + y \in P_i(x_1, \ldots, x_N)$ whenever y is strictly positive] and the positive cone L^+ has a non-empty interior, then extremely desirable commodities exist automatically. Indeed, let v be any vector in the interior of L^+ and choose a positive number μ such that the ball $B = \{w \in L; \|v - w\| < \mu\}$ is contained in the interior of L^+. Now, if $\|\sigma\| < t\mu$ then $\|t^{-1}\sigma\| < \mu$ so $tv - \sigma = t(v - t^{-1}\sigma)$ belongs to the interior of L^+. (for $t > 0$). Strict monotonicity now implies that $x_i + tv - \sigma \in P_i(x_1, \ldots, x_N)$. (Informally, $x_i + tv - \sigma$ is better than x_i because it is strictly bigger.) Since the positive cone L^+ has a non-empty interior for every finite-dimensional space L, and for l_∞ and L_∞, requiring existence of extremely desirable commodities imposes no additional restriction in these cases.

Extreme desirability may be given a very natural geometric interpretation. Fix a vector v in L^+ and a positive number μ, and let C be the open cone

$$C = \{tv - \sigma : 0 < t \leqq 1, \sigma \in L, \|\sigma\| < t\mu\}.$$