We shall say that the economy $\mathcal{E}$ is *irreducible* if: whenever I and J are non-empty sets of agents with $I \cap J = \emptyset$ and $I \cup J = \{1, \ldots, N\}$, and $(x_1, \ldots, x_N)$ is an allocation such that $\sum_{i=1}^{N} x_i = \sum_{i=1}^{N} e_i$, then there is an agent $m \in I$, an agent $n \in J$ and a vector $\zeta \in L$ with $0 \leqq \zeta \leqq e_n$ and $x_m + \zeta \in P_m(x_1, \ldots, x_N)$. [See McKenzie (1959).]

We will frequently refer to a vector $x \in L^+$ as a *commodity bundle.* We should caution the reader that, in our abstract framework, *there are no pure commodities.*

Finally, we make one comment about our use of Banach lattices as commodity spaces. It might seem more natural (and less restrictive) to use ordered Banach spaces, rather than lattices, as commodity spaces. However, many economic ideas lose their natural meanings if the lattice structure is missing. Suppose for example that the economy has two agents with initial endowments e_1 and e_2, and we consider the meaning of the statement 'the (positive) commodity bundle b is part of the aggregate initial endowment'. Presumably this should mean $0 \leqq b \leqq e_1 + e_2$. On the other hand, we should also like it to have the meaning that the whole bundle b is the sum of two parts, one of which is owned by each agent. In other words, there should exist bundles b_1, b_2 with $0 \leqq b_1 \leqq e_1, 0 \leqq b_2 \leqq e_2$ and $b = b_1 + b_2$. Unfortunately, if the commodity space *is not a lattice*, these two statements are *not equivalent.* On the other hand, if the commodity space *is a lattice*, these two statements *are equivalent* (this is just the Riesz Decomposition Property).

3. Preferences

The purpose of this section is to discuss in detail our key assumptions on preferences and their meaning. Throughout the remainder of this section, we let

$$\mathcal{E} = \{(X_1, P_i, e_i): i = 1, 2, \ldots, N\}$$

be an economy in the Banach lattice L. We will assume that each of the consumption sets X_i coincides with the positive cone L^+ of L (in section 8 we discuss ways in which this assumption can be weakened) and that $x_i \notin \operatorname{con} P_i(x_1, \ldots, x_N)$ for each agent i and each $(x_1, \ldots, x_N) \in \prod_{j=1}^{N} X_j$.

Fix an agent i, a vector $v \in L^+$ and an allocation $x = (x_1, \ldots, x_N) \in \prod X_j$. Let $\Gamma_v^i(x)$ denote the set of non-negative real numbers μ such that:

$$x_i + tv - \sigma \in P_i(x_1, \ldots, x_N) \quad \text{whenever} \quad 0 < t \leqq 1,$$

$$\sigma \leqq x_i + tv \quad \text{and} \quad \|\sigma\| < t\mu.$$