We frequently refer to a vector $(x_1,\dots,x_N)\in\prod X_j$ as an *allocation.* The interpretation of preferences which we have in mind is that $y_i\in P_i(x_1,\dots,x_N)$ means that agent i strictly prefers y_i to x_i if the (given) components of other agents are fixed; this is the usual way to allow for interdependent preferences. Notice that preferences need not be transitive or complete or convex. However, in all our results we shall asume that $x_i\notin \operatorname{con} P_i(x_1,\dots,x_N)$ for all $(x_1,\dots,x_N)\in\prod X_j$ (con A always denotes the convex hull of the set A); in particular, $x_i\notin P_i(x_1,\dots,x_N)$ so P_i is *irreflexive.*

The graph of the correspondence P_i is a subset of $\prod_{j=1}^N X_j\times X_i$. If τ is a topology on L, we shall say that P_i is (τ, norm)-*continuous* if the graph of P_i is an open set of the product $\prod_{j=1}^N X_j\times X_i$, where we endow each of the first N factors with the topology τ and the last factor X_i with the norm topology (product spaces will always be given the product topology). This is equivalent to saying that if $y_i\in P_i(x_1,\dots,x_N)$ then there are relatively τ-open neighborhoods U_j of x_j in X_j and relatively norm-open neighborhood V_i of y_i in X_i such that $\bar{y}_i\in P_i(\bar{x}_1,\dots,\bar{x}_N)$ whenever $\bar{y}_i\in V_i$ and $\bar{x}_j\in U_j$ for each $j=1,2,\dots,N$. Mixed continuity is common in infinite-dimensional settings; see Bewley (1972) for example. The topology τ we shall use will be different in different settings; we refer to section 4 for further discussion.

A *price* is a continuous linear functional π on L (i.e., $\pi\in L'$). By an *equilibrium* for the economy $\mathcal{E}$ we mean an $(N+1)$-tuple $(x_1,\dots,x_N;\pi)$ where $x_i\in X_i$ for each i and π is a non-zero price, such that

(i) $\sum_{i=1}^N x_i=\sum_{i=1}^N e_i$,
(ii) $\pi(x_i)=\pi(e_i)$ for each i,
(iii) if $y_i\in P_i(x_1,\dots,x_N)$ then $\pi(y_i)>\pi(e_i)$ (for each i).

(Notice that we do not require prices to be positive and that we treat *exact* equilibria rather than free disposal equilibria.) A *quasi-equilibrium*[3] is an $(N+1)$-tuple $(x_1,\dots,x_N;\pi)$ where $x_i\in X$ for each i, and π is a non-zero price, such that (i), (ii) above and the following hold:

(iii′) if $y_i\in P_i(x_1,\dots,x_N)$ then $\pi(y_i)\geqq\pi(e_i)$ (for each i).

We have restricted our attention to continuous prices because that seems economically natural. However, in Yannelis–Zame (1984) we show that, in the context we consider in this paper, discountinuous prices can safely be ignored. That is, allocations which can be supported in equilibrium by discontinuous prices can also be supported in equilibrium by continuous prices.

[3]Strictly speaking, this defines a compensated equilibrium, rather than a quasi-equilibrium. However, in the presence of our other assumptions, these two notions are equivalent.