

the allocation of resources over an infinite time horizon,<sup>1</sup> and the Lebesgue space  $L_\infty$  of bounded measurable functions on a measure space to model uncertainty. Duffie and Huang (1985) use the space  $L_2$  of square-integrable functions on a measure space to model the trading of long-lived securities over time. Finally, Mas-Colell (1975) and Jones (1983) use the space  $M(\Omega)$  of measures on a compact metric space to model differentiated commodities.

This paper establishes a very general result on the existence of competitive equilibria for exchange economies (with a finite number of agents) with an infinite-dimensional commodity space. The commodity spaces we treat are Banach lattices, and include all the sequence spaces  $l_p$  ( $1 \leq p \leq \infty$ ), all the Lebesgue spaces  $L_p$  ( $1 \leq p \leq \infty$ ) and the space  $M(\Omega)$  of measures. Thus we allow for commodity spaces which are general enough to include most of the spaces used in economic analysis. Moreover, we allow for preferences which may not be monotone, transitive or complete; preferences may even be interdependent. Since preferences need not be monotone, we allow for prices which need not be positive, and obtain an exact equilibrium rather than a free-disposal equilibrium.

The central assumption we make is that preferences satisfy the non-transitive version of a condition used by Mas-Colell (1983), which he called 'uniform properness' [and which is, in turn, related to a condition used by Chichilnisky and Kalman (1980)]. Informally, preferences satisfy this condition if there is one commodity bundle which is a uniformly good substitute for any other commodity bundle (in appropriate quantities). This assumption is quite weak; it is automatically satisfied, for example, whenever preferences are monotone and the positive cone has a non-empty interior. (This includes all finite-dimensional spaces and the infinite-dimensional spaces  $l_\infty$  and  $L_\infty$ .) It also admits many natural economic interpretations; for example, in infinite time horizon models it corresponds to the assumption that agents do not over-emphasize the future.

Our work is closely related to the work of Mas-Colell (1983), in the sense that our crucial assumption is analogous to his. However, Mas-Colell's argument [which is related to an idea of Magill (1981) and Negishi (1960)] depends crucially on completeness and transitivity of preferences. On the other hand, the arguments of Bewley (1972), which have been generalized by Florenzano (1983), Toussaint (1984), Khan (1984) and others, depend crucially on monotonicity (or free disposability) and on the assumption that the positive cone of the commodity space has a non-empty interior. (Of the commodity spaces mentioned previously, only  $l_\infty$  and  $L_\infty$  enjoy this pro-

<sup>1</sup>The space  $l_1$  of summable sequences can also be used for such a model. Our choice between  $l_\infty$  and  $l_1$  should be based on the sort of resources we have in mind. If we are considering a renewable resource (such as food) we should use  $l_\infty$ , since the finiteness of the earth places an upper bound on the amount available in any time period. If we are considering a non-renewable resource (such as oil), it seems more appropriate to use  $l_1$ , since not only the amount available in each period, but also the sum total available throughout time is (presumably) bounded.