

therefore the aggregate demand set may not be convex. Of course such a problem does not arise in Schmeidler's framework since with an atomless measure space of agents and finitely many commodities the aggregate demand set is always convex as a consequence of the Lyapunov Theorem. [This is also the case in Rustichini-Yannelis (1990) where the economy has "many more" agents than commodities and there is a convexifying effect on aggregation.]

Remark 7.2. If the convexity assumptions on preferences is relaxed from our model, once we assume that the measure space of agents is atomless, then we can easily prove the existence of an approximate or δ -competitive equilibrium. In particular, one can convexify the demand set $D(t, p)$ by taking its closed convex hull, i.e., $\overline{\text{con}}D(t, p)$. [Notice that for each fixed $t \in T$, $\overline{\text{con}}D(t, p)$ is u.s.c. and for each fixed $p \in \Delta$, $\overline{\text{con}}D(\cdot, p)$ has a measurable graph.] Note that by Theorem 1 in Khan (1986) [see also Yannelis (1990a, Theorem 6.3)] we have that

$$\text{cl} \int_T D(t, p) d\mu(t) = \int_T \overline{\text{con}}D(t, p) d\mu(t).$$

Proceeding now as in the proof of the Auxiliary Theorem one can easily show that $x(t) \in D(t, p)$ for almost all t in T and $\|\int_T x(t) d\mu(t) - \int_T e(t) d\mu(t)\| < \delta$. Note that now the completeness assumption on preferences is not needed (recall Remark 7.1).

Remark 7.3. The space $C(X)$, i.e., the space of continuous functions on the compact metric space X with the sup norm is an ordered separable Banach space whose positive cone has a nonempty norm interior. Hence, the Main Theorem covers $C(X)$. It is important to note that in this space even if the set of agents is finite one cannot relax the bound from the consumption sets. In particular, since order intervals are not compact in any topology, one cannot conclude that the set of all feasible allocations (which always lie in an order interval) is compact. Finally, it is important to note that our Main Theorem covers $L_\infty(\Omega)$, i.e., the space of essentially bounded measurable functions on the measure space Ω , with the sup norm. This is due to the fact that weakly compact subsets of L_∞ are norm separable [see for instance Diestel-Uhl (1977, Theorem 13, p. 252)].