for each fixed $p \in \Delta$, $D(\cdot, p)$ has a measurable graph and is nonempty valued, the first conclusion of Corollary 4.1 assures that $\int D(\cdot, p)$ is nonempty and therefore, $\zeta(p)$ is nonempty for each $p \in \Delta$. Since for each fixed $t \in T$, $D(t, \cdot)$ is u.s.c. and $D(\cdot, \cdot)$ is convex, closed valued and $D(t,p) \subset X(t)$ for all $t \in T$, where $X : T \to 2^{E_+}$ is integrably bounded, norm compact, convex, nonempty valued, it follows from the second conclusion of Corollary 5.1 that $\int D(t, \cdot)$ is weakly u.s.c. and so ζ is weakly u.s.c. as well. Finally, it follows from Lemma 5.1 that $\int_T D(t,p)d\mu(t)$ is weakly compact, and hence, $\zeta(p)$ is weakly compact for each $p \in \Delta$. Consequently, ζ satisfies all the assumptions of the Main Lemma and therefore that there exist $(\bar{p}, \bar{x})$ such that $\bar{x} \in \zeta(\bar{p})$ and $\bar{x} \leq 0$, i.e., $\bar{x} = \int_T f(t)d\mu(t) - \int_T e(t)d\mu(t) \leq 0$ and $f(t) \in D(t,p)$ for almost all $t \in T$. Hence $(\bar{p}, f)$ is a competitive equilibrium and this completes the proof of the Auxiliary Theorem.

6. Proof of the Main Theorem

Let $\mathcal{F}$ be a family of all nonempty, norm compact, convex subsets of E_+ containing the initial endowments. For each $F \in \mathcal{F}$ define the consumption correspondence $X^F : T \to 2^{E_+}$ by

$$X^F = F \cap X(t).$$

Moreover, for each $F \in \mathcal{F}$ let $\succsim_t^F$ be the preference relation of agent t induced on F. Let $S^1_{X^F} = \{x \in L_1(\mu, E_+) : x(t) \in X^F(t)$ for almost all t in $T\}$.

We now have a truncated economy $\mathcal{E}^F = [(T, \tau, \mu), X^F, \succsim^F, e]$ which is easily seen that satisfies all the assumptions of the Auxiliary Theorem. Consequently, a competitive equilibrium in $\mathcal{E}^F$ exists, i.e., there exist (p_F, x_F), $p_F \in E^*_+/\{0\}$, $x_F \in S^1_X F$ such that:

(i) $x_F(t) \in D(t, p_F)$ for almost all t in T, and
(ii) $\int_T x_F(t)d\mu(t) \leq \int_T e(t)d\mu(t)$.

Since for each X is weakly compact, nonempty, and convex valued, by Diestel's Theorem, S^1_X is weakly compact in $L_1(\mu, E_+)$. Observe that for each $F \in \mathcal{F}$, $x_F \in A = \{y \in S^1_X : \int_T y(t)d\mu(t) \leq \int_T e(t)d\mu(t)\}$. It can be easily checked that A is convex and norm closed and as a consequence