Corollary 5.1 can be also obtained as a corollary of the infinite dimensional version of Fatou's Lemma proven in Yannelis (1988). For a further discussion on this see Yannelis (1990a).

The following result proved in Yannelis (1988, Lemma 3.1) is the infinite dimensional extension of Theorem 4 in Aumann (1965).

Lemma 5.1. *Let* (T, τ, μ) *be a complete finite measure space and* Y *be a separable Banach space. Let* $\phi : T \to 2^Y$ *be a closed, convex valued correspondence such that* $\phi(t) \subset X(t)$ *for all* $t \in T$, *where* $X : T \to 2^Y$ *is an integrably bounded, nonempty, weakly compact, convex valued correspondence. Then,*

$$\int_T \phi(t) d\mu(t) \quad \textit{is weakly compact.}$$

Notice that Aumann (1965) does not require $\phi(\cdot)$ to be convex valued. However, it can be easily shown that the above result is false without the convex valuedness of ϕ [see Rustichini (1989) or Yannelis (1990a)].

We now state a recent result proved in Khan-Vohra (1985, Theorem B, p. 331).

Lemma 5.2. *Let* $\{z_k : k \in K\}$ *be a net in* B, *where* B *is a weakly compact subset of a Banach space, and suppose that* z_k *converges weakly to* z. *Then we can extract a sequence* $\{z_n : n = 1, 2, \ldots\}$ *from the net* $\{z_k : k \in K\}$ *which converges weakly to* z.

With all these preliminary results out of the way, we can now complete the proof of the Auxiliary Theorem.

Let $\Delta = \{p \in E_+^* : p \cdot u = 1\}$ be the price space. It follows from Alaoglu's Theorem [Jameson (1970, p. 123)] that Δ is weak* compact. Moreover, since E is a separable Banach space, Δ is metrizable, [Dunford-Schwartz (1958, p. 426)]. For $p \in \Delta$ and $t \in T$, let the budget set be $B(t,p) = \{x \in X(t) : p \cdot x \leq p \cdot e(t)\}$. Since for each $t \in T$, $X(t)$ is norm compact and Δ is weak* compact, the bilinear form $(p, x) \to p \cdot x$ is jointly continuous [see for instance Yannelis-Zame (1986) Lemma A, p. 107]. Hence, it follows from assumption (3.4) that for each fixed $t \in T$, $B(t, \cdot)$ is continuous and a standard argument can be adopted to show that for each fixed $t \in T$, $D(t, \cdot)$ is u.s.c. in the sense that the set $\{p \in \Delta : D(t,p) \subset V\}$ is weak* open in Δ for every norm space subset V of E_+.