$z_n(t)$ converges in norm to $y(t)$ (otherwise pass to a subsequence) for all $t \in T/S$, where $S \subset T$, $\mu(S) = 0$. Fix t in T/S. Since for each fixed $t \in T$, $D(t, \cdot)$ is norm for u.s.c., for every $\delta > 0$ we can find n, $(n = 1, 2, \ldots)$ such that for all $n_0 \geq n$ we have $D(t, p_{n_0}) \subset D(t,p) + \delta B$, (where B is the open unit ball in Y). Hence

$$\text{con} \bigcup_{n_0 \geq n} D(t, p_{n_0}) \subset D(t,p) + \delta B \Rightarrow z_n(t) \in D(t,p) + \delta B$$
$$\Rightarrow y(t) \in D(t,p) + \delta B.$$

By letting δ go to zero, we conclude that $y(t) \in D(t,p)$. Since $t \in T/S$ was arbitrary, $y(t) \in D(t,p)$ for almost all t in T. Finally since $D(t,p) \subset X(t)$ for all $t \in T$ and $D(\cdot, \cdot)$ is integrably bounded, we can conclude that $y \in \phi(p)$. This completes the proof of the Theorem.

Corollary 5.1. *Let $D : T \times P \to 2^Y$ be a correspondence satisfying all the assumptions of Theorem 5.1. Then,*

(i) $\int D(\cdot, p)$ is nonempty, and
(ii) $\int D(t, \cdot)$ is weakly u.s.c.

Proof. (i) Since for each $p \in P$, $\psi(p) \neq \phi$, it follows that $\int D(\cdot, p)$ is nonempty.

(ii) We now show that $\int D(t, \cdot)$ is weakly u.s.c. Define $\psi : P \to 2^{S^1_X}$ by $\psi(p) = \{y \in S^1_X : y(t) \in D(t,p)$ for almost all t in $T\}$. Let $f : S^1_X \to Y$ be a mapping defined by $f(\psi(p)) = \int D(t,p)$. Clearly f is norm continuous and linear. It is a standard result [see for instance Aliprantis-Burkinshaw (1985, Theorem 9.16, p. 139) or Dunford-Schwartz (1958, Theorem 15, p. 422)] that f is also weakly continuous. By Theorem 5.1, ψ is weakly u.s.c. and so is $f(\psi)$. Hence, $\int D(t, \cdot)$ is weakly u.s.c. and this completes the proof of the Corollary.

Remark 5.1. The separability assumption on the measure space (T, τ, μ) in Theorem 5.1 and Corollary 5.1 is not needed provided the reader follows the argument in Yannelis (1990a, Theorem 5.4 and Remark 5.1).

Observe that conclusions (i) and (ii) of the above corollary have been proved by Aumann (1965, 1976) for $Y = \mathbb{R}^\ell$. Hence the above corollary may be seen as an extension of Aumann's result. Conclusion (ii) of