Normed by the functional $\|\cdot\|$ above, $L_1(\mu, X)$ becomes a Banach space [(see Diestel-Uhl (1977, p. 50)]. A correspondence $\phi : T \to 2^X$ is said to have a *measurable graph* if $G_\phi \in \tau \otimes \beta(X)$ where $\beta(X)$ denotes the Borel σ-algebra on X and $\otimes$ denotes product σ-algebra. The correspondence $\phi : T \to 2^X$ is said to be *lower measurable* if for every open subset V of X the set $\{t \in T : \phi(t) \cap V \neq \emptyset\}$ belongs to τ. The correspondence $\phi : T \to 2^X$ is said to be *integrably bounded* if there exists a map $h \in L_1(\mu, \mathbb{R})$ such that for almost all $t \in T$, $\sup\{\|x\| : x \in \phi(t)\} \leq h(t)$. A *measurable selection* for the correspondence $\phi : T \to 2^X$ is a measurable function $f : T \to X$ such that $f(t) \in \phi(t)$ for almost all $t \in T$. A well-known result of Aumann (1967) says that if ϕ is a correspondence from a complete finite measure space to a separable metric space such that ϕ has a measurable graph and it is nonempty valued, then ϕ has a measurable selection. Following Aumann (1965) we now define the notion of the Aumann integral. Let T be a finite measure space, X be a Banach space and $\phi : T \to 2^X$ be a correspondence. We denote by S^1_ϕ the set of all X-valued Bochner integrable selections for $\phi(\cdot)$, i.e., $S^1_\phi = \{x \in L_1(\mu, X) : x(t) \in \phi(t)$ for almost all $t \in T\}$. In the sequel we will call the above set, *the set of integrable selections*. We are now ready to define the integral of the correspondence $\phi(\cdot)$ as follows:

$$\int_T \phi(t) d\mu(t) = \left\{ \int_T x(t) d\mu(t) : x(\cdot) \in S^1_\phi \right\}.$$

We will denote the above integral as $\int \phi(\cdot)$, and call it the *Aumann integral*. We now state a result which will play a crucial role in the sequel. This is *Diestel's Theorem* [Diestel (1977)], which says that if $K : T \to 2^Y$ (here T is a finite measure space and Y is a separable Banach space) is an integrably bounded, convex, nonempty weakly compact valued correspondence, then S^1_K is weakly compact in $L_1(\mu, Y)$.

3. The Main Theorem

3.1 The Model. We now turn to the main result of the paper, i.e., the existence of a competitive equilibrium in economies with infinitely many commodities and agents.