2.2 Definitions. Let X, Y be two topological spaces. A set-valued function (or correspondence) $\phi : X \to 2^Y$ is said to be *upper semicontinuous* (u.s.c.) if the set $\{x \in X : \phi(x) \subset V\}$ is open in X for every open subset V of Y. Throughout the paper we will consider the setting where X is a metric space and Y is a Banach space. In this setting we will say that ϕ is *norm* u.s.c., if the set $\{x \in X : \phi(x) \subset V\}$ is open in X for every norm open subset V of Y. Furthermore, we will say that ϕ is *weakly* u.s.c., if the set $\{x \in X : \phi(x) \subset V\}$ is open in X for every weakly open subset V of Y.

Let X and Y be sets. The *graph* of the correspondence $\phi : X \to 2^Y$ is denoted by $G_\phi = \{(x, y) \in X \times Y : y \in \phi(x)\}$.

We now define the notion of a Bochner integrable function. Let (T, τ, μ) be a finite measure space, and X be a Banach space. A function $f : T \to X$ is called *simple* if there exist $x_1, x_2, \ldots, x_n$ in X and $\tau_1, \tau_2, \ldots, \tau_n$ in τ such that $f = \sum_{i=1}^n x_i \chi_{\tau_i}$, where

$$\chi_{\tau_i}(t) = \begin{cases} 1 & \text{if } t \in \tau_i \\ 0 & \text{if } t \notin \tau_i. \end{cases}$$

A function $f : T \to X$ is said to be *μ-measurable* if there exists a sequence of simple functions $f_n : T \to X$ such that $\lim_{n\to\infty} \|f_n(t) - f(t)\| = 0$ for almost all $t \in T$. A μ-measurable function $f : T \to X$ is said to be *Bochner integrable* if there exists a sequence of simple functions $\{f_n : n = 1, 2, \ldots\}$ such that

$$\lim_{n\to\infty} \int_T \|f_n(t) - f(t)\| d\mu(t) = 0.$$

In this case we define for each $E \in \tau$ the integral to be $\int_E f(t) d\mu(t) = \lim_{n\to\infty} \int_E f_n(t) d\mu(t))$. It is a standard result [see Diestel-Uhl (1977, Theorem 2, p. 45)] that, if $f : T \to X$ is a μ-measurable function then ϕ is Bochner integrable if and only if $\int_T \|f(t)\| d\mu(t) < \infty$.

Let (T, τ, μ) be a complete finite measure space, i.e., μ is a real-valued, non-negative countably additive measure defined on a complete σ-field τ of subsets of T such that $\mu(T) < \infty$. Let X be a Banach space. We denote by $L_1(\mu, X)$ the space of equivalence classes of X-valued Bochner integrable functions $f : T \to X$ normed by

$$\|f\| = \int_T \|f(t)\| d\mu(t).$$