Note that $\int_T \tilde{y}^k - \int_T \tilde{e}^k \notin -C$ and therefore (6.4) holds.

We can now separate the convex non-empty set $\mathrm{cl}\int \phi - \int e$ from the convex non-empty set $-C$. Proceeding as in the proof of Theorem 4.1, we can now complete the proof.

Proof of Claim 6.1. We will argue by contradiction, and for notational convenience we will drop the index k.

Suppose that Claim 6.1 is false, then

$$\sum_{i=1}^{m} y_i\xi - \sum_{i=1}^{m} e_i\xi \in -\alpha(v+U),$$

and therefore

$$\sum_{i=1}^{m} y_i + w - u = \sum_{i=1}^{m} e_i, \quad \text{where} \quad w = \frac{\alpha}{\xi} v, \quad u \in \frac{\alpha}{\xi} U. \tag{6.8}$$

Note that without loss of generality we may assume that $u \geqq 0$ [otherwise, since $u = u^+ - u^-$, we may define $\hat{y}_i = y_i + u^+/mj$ then $\hat{y}_i \geqq 0$ and $u^- \in \alpha U$ (recall that U can be assumed to be solid), $\hat{y}_i >_t x(t)$ for all $t \in T_i$ and all i and one can proceed by substituting y_i for $\hat{y}_i$].

It follows from (6.8) that for any m-tuple $(\theta_1, \ldots, \theta_m)$, $\theta_i \geqq 0$ $(i = 1, \ldots, m)$, $\sum_{i=1}^m \theta_i = 1$, we have that

$$\sum_{i=1}^{m} (y_i + \theta_i w) - u = \sum_{i=1}^{m} e_i \geqq 0,$$

and therefore

$$u \leqq \sum_{i=1}^{m} (y_i + \theta_i w). \tag{6.9}$$

Applying the Riesz Decomposition Property in (6.9) we obtain $u_1, \ldots, u_m$ in E_+ such that

$$\sum_{i=1}^{m} u_i = u, u_i \leqq y_i - \theta_i w \quad \text{for} \quad \text{all } i. \tag{6.10}$$

(It is easy to see that the proof of the Riesz Decomposition Property provides an algorithm to choose in a unique way the u_i's above). For each i, define $\tilde{y}_i: [0,1] \to E_+$ by $\tilde{y}_i(\theta) = y_i + \theta_i w - u_i$. Moreover, for each i set $F_i(\tilde{y}_i(\theta)) = \mathrm{dist}(\tilde{y}_i(\theta), C + y_i) = \delta_i(\theta)$. Let $\Delta = \{q \in R^m_+ : \sum_{i=1}^m q_i = 1\}$. Define the continuous mapping