said to be an extremely desirable commodity with respect to U, if for each $x \in E_+$ and each $t \in T$, we have $y >_t x$ whenever y is an element of $(C+x) \cap E_+$.

Let δ_i, $i=1,\dots,n$ be positive real numbers with $\sum_{i=1}^n \delta_i = 1$.

A.11. Let U of A.10 satisfy the following condition: if $x_i \in E_+$, $x_i \notin \delta_i U$, $i=1,2,\dots,n$, then $\sum_{i=1}^n x_i \notin \sum_{i=1}^n \delta_i U = U$.

We note that in assumption A.11 the additivity condition only concerns the neighborhood of the extremely desirable commodity, not the commodity space. To clarify this point we shall consider a specific example. Consider the space $L_p(\Omega)$, $1 \leqq p < \infty$ where Ω is a finite separable measure space. From Holder's inequality, for $f \in L_p(\Omega)$ we have that $\|f\|_1 \leqq C\|f\|_p$ for some constant C depending only on p and Ω. Suppose now that assumption A.10 is satisfied with U containing a neighborhood U' of the form:

$$U' \equiv \{f \in L_p : \|f\|_1 < \varepsilon\}.$$

Then U' is open in L_p because of the inequality mentioned above. Moreover A.11 is also satisfied. Roughly speaking we require the neighborhood U' of the extremely desirable commodity to be 'large' in the topology of E.

Finally we need:

A.12. For each $x \in E_+$, the sets $\{y \in E_+ : y >_t x\}$ and $\{y \in E_+ : x >_t y\}$ are norm open in E_+ for all $t \in T$.

We can now state the following result:

Theorem 6.1. Under assumptions A.2–A.12, $C(\varepsilon) = W(\varepsilon)$.

Proof. It can be easily shown that $W(\varepsilon) \subset C(\varepsilon)$. Hence, we will show that if $x \in C(\varepsilon)$, then for some price p, the pair (x,p) is a competitive equilibrium for ε.

Define the correspondence $\phi: T \to 2^{E_+}$ by

$$\phi(t) = \{z \in E_+ : z >_t x(t)\} \cup \{e(t)\}. \tag{6.1}$$

Let C be the open cone spanned by the set $v+U$ given by assumptions A.10–A.11, i.e., $C = \operatorname{span}\{0, v+U\} \equiv \bigcup_{\alpha>0} \alpha(v+U)$. We claim that

$$\operatorname{cl}\left(\int \phi - \int e\right) \cap -C = \varnothing, \tag{6.2}$$

or equivalently,