Then $u(\tilde{x}(t)) \geqq u(e)$ for all $t \in T$ and $u(\tilde{x}(t)) > u(e)$ for all $t \in S$, contradicting the fact that e is Pareto optimal.

It follows from (5.2) and (5.3) that $C(\varepsilon) = \{e\}$ and this completes the proof of the fact that $C(\varepsilon) \neq \varnothing$ and $W(\varepsilon) = \varnothing$.

Since Example 5.1 satisfies all the conditions of Theorem 4.1 except assumption A.1 (note that $\operatorname{int} l_2^+ = \varnothing$), we can conclude that if positive results are to be obtained in spaces whose positive cone has an empty norm interior some additional assumption needs to be imposed. The additional assumption we impose is that of an extremely desirable commosity introduced in Yannelis and Zame (1986) [which is related to the assumption of proper preferences introduced by Mas-Colell (1986)].

6. Core–Walras equivalence in separable Banach lattices whose positive cone has an empty interior

We begin by defining the notion of an extremely desirable commodity. Let E be a Banach lattice and denote its positive cone (which may have an empty norm interior) by E_+. Let $v \in E_+, v \neq 0$. We say that $v \in E_+$ is an *extremely desirable commodity* if there exists an open neighborhood U such that for each $x \in E_+$ and each $t \in T$, we have that $x + \alpha v - z >_t x$ whenever $\alpha > 0$, $z \leqq x + \alpha v$ and $z \in \alpha U$. In other words, v is extremely desirable if an agent would prefer to trade any commodity bundle z for an additional increment of the commodity bundle v, provided that the size of z is sufficiently small compared to the increment of v. The above notion has a natural geometric interpretation. In particular, let $v \in E_+$, $v \neq 0$, U be an open neighborhood and define the open cone C as follows:

$$C = \{\alpha v - z : \alpha > 0, z \in E, z \in \alpha U\}.$$

The bundle v is said to be an extremely desirable commodity, if for each $x \in E_+$, and each $t \in T$ we have $y >_t x$ whenever y is an element of $(C + x) \cap E_+$. This implies that v is an extremely desirable commodity if for each $x \in E_+$ we have that $((-C + x) \cap E_+) \cap \{y : y >_t x\} = \varnothing$, or equivalently $-C \cap \{y - x \in E_+ : y >_t x\} = \varnothing$.

Recall that if the preference relation $>_t$ is monotone and $\operatorname{int} E_+ \neq \varnothing$, then the assumption of an extremely desirable commodity is automatically satisfied [see for instance Yannelis and Zame (1986)].

We now state our assumptions:

A.9. E is any separable Banach lattice.

A.10. (*Extremely desirable commodity*). Let $v \in E_+/\{0\}$ and U be an open convex neighborhood. Let C be the cone spanned by $v + U$. The bundle v is