We now show that μ-a.e. $p\cdot x(t)=p\cdot e(t)$. First note that if follows directly from (4.8) that $p\cdot x(t)\geqq p\cdot e(t)$ μ-a.e. If now $p\cdot x(t)>p\cdot e(t)$ for all $t\in S$, $\mu(S)>0$, then

$$p\cdot\int_T x=p\cdot\int_{T\setminus S} x+p\cdot\int_S x$$

$$>p\cdot\int_{T\setminus S} e+p\cdot\int_S e=p\cdot\int_T e,$$

contradicting $\int_T x=\int_T e$, since $p\geqq 0$, $p\neq 0$.

To complete the proof we must show that $x(t)$ is maximal in the budget set $\{z\in E_+ : p\cdot z\leqq p\cdot e(t)\}$ μ-a.e. The argument is now routine. Since $\int_T e$ is strictly positive (assumption A.4) it follows that $\mu(\{t: p\cdot e(t)\})>0$, for if $p\cdot e(t)=0$ μ-a.e., then $p\cdot\int_T e=0$ contradicting the fact that $\int_T e$ is strictly positive since $p\geqq 0$, $p\neq 0$.

Thus, we can safely pick an agent t with positive income, i.e., $p\cdot e(t)>0$. Since $p\cdot e(t)>0$ there exists an allocation x' such that $p\cdot x'<p\cdot e(t)$. Let y be such that $p\cdot y\leqq p\cdot e(t)$ and let $y(\lambda)=\lambda x'+(1-\lambda)y$ for $\lambda\in(0,1)$. Then for any $\lambda\in(0,1)$, $p\cdot y(\lambda)<p\cdot e(t)$ and by (4.8) $y(\lambda)\not>_t x(t)$. It follows from the norm continuity of $>_t$ (assumption A.5) that $y\not>_t x(t)$. This proves that $x(t)$ is maximal in the budget set of agent t, i.e., $\{w: p\cdot w\leqq p\cdot e(t)\}$. This, together with the monotonicity of preferences (assumption A.8) implies that prices are strictly positive, i.e., $p\gg 0$. Indeed, if there exists $v\in E_+\setminus\{0\}$ such that $p\cdot v=0$ then $p\cdot(x(t)+v)=p\cdot x(t)=p\cdot e(t)$ and by monotonicity $x(t)+v>_t x(t)$ contradicting the maximality of $x(t)$ in the budget set.

Thus $p\gg 0$ and $x(t)$ is maximal in the budget set whenever $p\cdot e(t)>0$. Consider now an agent t with zero income, i.e., $p\cdot e(t)=0$. Since $p\gg 0$ his/her budget set $\{z: p\cdot z=0\}$ consists of zero only, and moreover, $p\cdot x(t)=p\cdot e(t)=0$. Hence, $x(t)=0$ for almost all $t\in T$, with $p\cdot e(t)=0$; i.e., zero in this case is the maximal element in the budget set. Consequently, (p,x) is a competitive equilibrium for ε, and this completes the proof of Theorem 4.1.

5. The failure of the core–Walras equivalence in commodity spaces whose positive cone has an empty interior

In the previous section we showed that if the commodity space is an ordered separable Banach space E whose positive cone has a non-empty norm interior (i.e., $\operatorname{int} E_+\neq\varnothing$), then the standard assumptions (i.e., the assumptions of Theorem 4.1) guarantee core–Walras equivalence. We now show that if the assumption that the positive cone of the space E has a non-empty norm interior is dropped, then Theorem 4.1 fails. The following example will illustrate this.