$$\phi(t)=\{z\in E_+ : z >_t x(t)\} \cup \{e(t)\}. \tag{4.1}$$

We claim that:

$$\mathrm{cl}\left(\int_T \phi - \int_T e\right) \cap \operatorname{int} E_- = \varnothing, \tag{4.2}$$

or equivalently,[5]

$$\left(\int_T \phi - \int_T e\right) \cap \operatorname{int} E_- = \varnothing. \tag{4.3}$$

Suppose otherwise, i.e.,

$$\left(\int_T \phi - \int_T e\right) \cap \operatorname{int} E_- \neq \varnothing,$$

then there exists $v \in \operatorname{int} E_+$ such that

$$\int e - v \in \int \phi. \tag{4.4}$$

It follows from (4.4) that there exists a function $y: T \to E_+$ such that

$$\int_T y = \int_T e - v, \tag{4.5}$$

and $y(t)\in\phi(t)$ μ-a.e.

Let

$$S=\{t: y(t) >_t x(t)\}, \quad \text{and}$$

$$S'=\{t: y(t)=e(t)\}.$$

Since $\int y \neq \int e$ we have that $\mu(S)>0$. Define $\tilde{y}: S\to E_+$ by $\tilde{y}(t)=y(t)+v/\mu(S)$ for all $t\in S$. By monotonicity (assumption A.8) $\tilde{y}(t) >_t y(t)$. Since $y(t) >_t x(t)$ for all $t\in S$, by transitivity (assumption A.6) $\tilde{y}(t) >_t x(t)$ for all $t\in S$. Moreover, it can be easily seen that $\tilde{y}(\cdot)$ is feasible for the coalition S, i.e.,

[5]This is so since $\operatorname{int} E_-$ is an open set. In particular, if A and B are subsets of any topological space and B is open, then it can be easily seen that $A\cap B=\varnothing$ if and only if $\mathrm{cl}\, A\cap B=\varnothing$.